again for the true 2d difference, at the beginning of that class. Thus, the 10th root of 1,0002302850, or E, gives 1,000023026116 for F, or the 1st mean of the lowest class, therefore $F - 1 = r - 1 =$,000023026116, is its 1st difference, and the square of it is $(r-1)^2 =$,0000000005302 its 2d diff.; then is ,000023026116F^{10n} or ,000023026116E^n, the 1st difference, and ,0000000005302F^{20n} or ,0000000005302E^{2n} is the 2d difference, at the beginning of the nth class of decades. And this 2d difference is used as the constant 2d difference through all the 10 terms, except towards the end of the table, where the differences increase fast enough to require a small correction of the 2d difference, which Mr. Dodson effects by taking a mean 2d difference among all the 2d differences, in this manner; having found the series of 1st differences $(F-1).E^n$, $(F-1).E^{n+1}$, $(F-1).E^{n+2}$, &c, he takes the differences of these, and $\frac{1}{10}$ of them gives the mean 2d differences to be used, namely, $\frac{F-1}{10}(E^{n+1}-E^n)$, $\frac{F-1}{10}(E^{n+2}-E^{n+1})$, &c, are the mean 2d differences. And this is not only the more exact, but also the easier way. The common 2d difference, and the successive 1st differences, are then continually added, through the whole decade, to give the successive terms of the required progression.

TRACT XXII.

SOME PROPERTIES OF THE POWERS OF NUMBERS.

1. Of any two square numbers, at any distance from each other in the natural series of the squares 1^2, 2^2, 3^2, 4^2, &c, the mean proportional between the two squares, is equal to the less square plus its root multiplied by the difference of the roots, that is, by the distance in the series between the two square numbers, or by 1 more than the number of squares between them. The same mean proportional, is also equal to the greater of the two squares, minus its root the same

number of times taken. That is, $mn = mm + dm = nn - dn$; where d is $= n - m$, the distance between the two squares m^2, n^2. For, since $n = m + d$; multiply by m, then $mn = mm + md$, which is the first part of the proposition. Again, $m = n - d$; multiply this by n, then $mn = nn - nd$, which is the latter part.

2. An arithmetical mean between the two squares mm and nn, exceeds their geometrical mean, by half the square of the difference of their roots, or of their distance in the series. For, by the first section, $mn = mm + dm$, and also $mn = nn - dn$; add these two together, and the sums are $2mn = mm + nn - d(n - m) = mm + nn - dd$; divide by 2, then $mn = \frac{1}{2}mm + \frac{1}{2}nn - \frac{1}{2}dd$.

3. Of three adjacent squares in the series, the geometrical mean between the extremes, is less by 1 than the middle square. For, let the three squares be m^2, $(m+1)^2$, $(m+2)^2$; then the mean between the extremes, $m(m+2) = mm + 2m$ is $= (m + 1)^2 - 1$.

In like manner, the mean between the extremes, of any three squares, whose common distance or difference of their roots is d, is less than the middle square by the square of the distance dd.

4. The difference between the two adjacent squares mm, nn, or $nn - mm$, is $(m + 1)^2 - m^2 = 2m + 1$. In like manner, the difference between n^2 and the next following square p^2, or $p^2 - n^2$, is $2n + 1$; and so on. Hence, the difference of these differences, or the 2d difference of the squares, is $2(n - m) = 2$, which is constant, because $n - m = 1$. And thus, the 2d differences being constantly the number 2, all the first differences will be found by the continual addition of this number 2; and then the whole series of squares themselves will be found by the continual addition of the first differences. Thus, the

2d difs.	2,	2,	2,	2,	2,	2,	2,	2,	2,	2,	&c.
1st difs.	1,	3,	5,	7,	9,	11,	13,	15,	17,	19,	&c.
squares,	1,	4,	9,	16,	25,	36,	49,	64,	81,	100,	&c.

5. Again, if m^3, n^3, p^3, be three adjacent cubes; then $\left.\begin{array}{l} n^3 - m^3 = 3m^2 + 3m + 1 \\ p^3 - n^3 = 3n^2 + 3n + 1 \end{array}\right\}$; and the differences of these first differences is $3(n^2 - m^2) + 3(n - m) = 6(m + 1)$, the 2d difference. In like manner, the next 2d difference will be $6(n+1)$. Then the dif. of these 2d differences is $6(n - m) = 6$ the 3d difference, which therefore is constant. Now, supposing the series of cubes to begin from 0, the first of each of the several orders of differences will be found by making $m = 0$, in the general expression for each order: thus, $6(m+1)$ becomes 6 for the first of the 2d differences; and $3m^2 + 3m + 1$ becomes 1 for the first of the 1st differences. And hence is found all the others, as in this table.

3d difs.	6,	6,	6,	6,	6,	6,	6,	6,	6,	&c.
2d difs.	6,	12,	18,	24,	30,	36,	42,	48,	54,	&c.
1st difs.	1,	7,	19,	37,	61,	91,	127,	169,	217,	&c.
cubes	0,	1,	8,	27,	64,	125,	216,	343,	512,	&c.

And thus may all the powers of the series of natural numbers 1, 2, 3, 4, 5, &c, be found, by addition only, adding continually the numbers throughout the several orders of differences. And here it is remarkable, that the number of the orders of differences, will be the same as the index of the powers to be formed; that is, in the series of squares, there are two orders of differences; in the cubes, three; in the 4th powers, four, &c: or, which is the same thing, of the squares, the 2d differences are equal to each other; of the cubes, the 3d differences are equal; of the 4th power, the 4th diffs. are equal; &c. Further, the 2d diffs. in the squares are $1.2 = 2$; the 3d diffs. in the cubes $1.2.3 = 6$; the 4th diffs. in the 4th powers $1.2.3.4 = 24$; and so on. And from these properties were found, by continual additions only, all the series of squares and cubes in the table at the end of this volume, and in my large Table of the Products and Powers of Numbers, published in 1781, by the Board of Longitude.