dently Latin words, and seem to be intended for the Latin translations of the names by which the Arabians called these lines, or the numbers expressing the lengths of them.

And this conjecture has been confirmed and realised, by a reference to Golius's Lexicon of the Arabic and Latin languages. In consequence I find that the Arabic and Latin writers on trigonometry do both of them use those words in the same allegorical sense, the latter being the Latin translations of the former, and not the Arabic words corrupted. Thus, the true Arabic word to denote the trigonometrical sine is جيب, pronounced *Jeib*, (reading the vowels in the French manner), meaning *sinus indusii, vestisque*, the bosom part of the garment; the versed sine is سهم, *Sehim*, which is *sagitta*, the arrow; the arc is قوس, which is *arcus*, the arc; and the chord is وتر, *Vitr*, that is *chorda*, the chord.

TRACT XX.

HISTORY OF LOGARITHMS.

The trigonometrical canon, of natural sines, tangents, and secants, being now brought to a considerable degree of perfection; the great length and accuracy of the numbers, together with the increasing delicacy and number of astronomical problems, and spherical triangles, to the solution of which the canon was applied, urged many persons, conversant in those matters, to endeavour to discover some means of diminishing the great labour and time, requisite for so many multiplications and divisions, in such large numbers as the tables then consisted of. And their chief aim was, to reduce the multiplications and divisions to additions and subtractions, as much as possible.

For this purpose, Nicholas Raymer Ursus Dithmarsus invented an ingenious method, which serves for one case in the sines, namely, when radius is the first term in the proportion, and the sines of two arcs are the second and third terms; for he showed, that the fourth term, or sine, would be found by only taking half the sum or difference of the sines of two other arcs, which should be the sum and difference of the less of the two former given arcs, and the complement of the greater. This is no more, in effect, than the following well-known theorem in trigonometry: as half radius is to the sine of one arc, so is the sine of another arc, to the cosine of the difference minus the cosine of the sum of the said arcs. The author published this ingenious device, in 1588, in his "Fundamentum Astronomiæ." And three or four years afterwards it was greatly improved by Clavius, who adapted it to all proportions in the solution of spherical triangles, for sines, tangents, secants, versed sines, &c; and that whether radius be in the proportion or not. All which he explains very fully in lem. 53, lib. 1, of his treatise on the Astrolabe. See more on this subject in Longomont. Astron. Danica. pa. 7, et seq. This method, though ingenious enough, depends not on any abstract property of numbers, but only on the relations of certain lines, drawn in and about the circle; for which reason it was rather limited, and sometimes attended with trouble in the application.

After perhaps various other contrivances, incessant endeavours at length produced the happy invention of logarithms, which are of direct and universal application to all numbers abstractedly considered, being derived from a property inherent in numbers themselves. This property may be considered, either as the relation between a geometrical series of terms and a corresponding arithmetical one, or as the relation between ratios and the measures of ratios, which comes to much the same thing, having been conceived in one of these ways by some of the writers on this subject, and in the other by the rest of them, as well as in both ways at different times by the same writer. A succinct idea of this property, and of the probable reflections made on it by the first writers on logarithms, may be to the following effect:

The learned calculators, about the close of the 16th, and beginning of the 17th century, finding the operations of multiplication and division by very long numbers, of 7 or 8 places of figures, which they had frequently occasion to perform, in resolving problems relating to geography and astronomy, to be exceedingly troublesome, set themselves to consider, whether it was not possible to find some method of lessening this labour, by substituting other easier operations in their stead. In pursuit of this object, they reflected, that since, in every multiplication by a whole number, the ratio, or proportion, of the product to the multiplicand, is the same as the ratio of the multiplier to unity, it will follow that the ratio of the product to unity (which, according to Euclid's definition of compound ratios, is compounded of the ratios of the said product to the multiplicand and of the multiplicand to unity), must be equal to the sum of the two ratios of the multiplier to unity and of the multiplicand to unity. Consequently, if they could find a set of artificial numbers that should be the representatives of, or should be proportional to, the ratios of all sorts of numbers to unity, the addition of the two artificial numbers that should represent the ratios of any multiplier and multiplicand to unity, would answer to the multiplication of the said multiplicand by the said multiplier, or the sum arising from the addition of the said representative numbers, would be the representative number of the ratio of the product to unity; and consequently, the natural number to which it should be found, in the table of the said artificial or representative numbers, that the said sum belonged, would be the product of the said multiplicand and multiplier. Having settled this principle, as the foundation of their wished-for method of abridging the labour of calculations, they resolved to compose a table of such artificial numbers, or numbers that should be representatives of, or proportional to, the ratios of all the common or natural numbers to unity.

The first observation that naturally occurred to them in the pursuit of this scheme was, that whatever artificial numbers should be chosen to represent the ratios of other whole numbers to unity, the ratio of equality, or of unity to unity, must be represented by 0; because *that* ratio has properly no magnitude, since, when it is added to, or subtracted from, any other ratio, it neither increases nor diminishes it.

The second observation that occurred to them was, that any number whatever might be chosen at pleasure for the representative of the ratio of any given natural number to unity; but that, when once such choice was made, all the other representative numbers would be thereby determined, because they must be greater or less than that first representative number, in the same proportions in which the ratios represented by them, or the ratios of the corresponding natural numbers to unity, were greater or less than the ratio of the said given natural number to unity. Thus, either 1, or 2, or 3, &c, might be chosen for the representative of the ratio of 10 to 1. But, if 1 be chosen for it, the representatives of the ratios of 100 to 1 and 1000 to 1, which are double and triple of the ratio of 10 to 1, must be 2 and 3, and cannot be any other numbers; and, if 2 be chosen for it, the representatives of the ratios of 100 to 1 and 1000 to 1, will be 4 and 6, and cannot be any other numbers; and, if 3 be chosen for it, the representatives of the ratios of 100 to 1 and 1000 to 1, will be 6 and 9, and cannot be any other numbers; and so on.

The third observation that occurred to them was, that, as these artificial numbers were representatives of, or proportional to, ratios of the natural numbers to unity, they must be expressions of the numbers of some smaller equal ratios that are contained in the said ratios. Thus, if 1 be taken for the representative of the ratio of 10 to 1, then 3, which is the representative of the ratio of 1000 to 1, will express the number of ratios of 10 to 1 that are contained in the ratio of 1000 to 1. And if, instead of 1, we make 10,000,000, or ten millions, the representative of the ratio of 10 to 1, (in which case 1 will be the representative of a very small ratio, or *ratiuncula*, which is only the ten-millionth part of the ratio of 10 to 1, or will be the representative of the 10,000,000th root of 10, or of the first or smallest of 9,999,999 mean proportionals interposed between 1 and 10), the representative of the ratio of 1000 to 1, which will in this case be 30,000,000, will express the number of those *ratiunculæ*, or small ratios of the 10,000,000th root of 10 to 1, which are contained in the said ratio of 1000 to 1. And the like may be shown of the representative of the ratio of any other number to unity. And therefore they thought these artificial numbers, which thus represent, or are proportional to, the magnitudes of the ratios of the natural numbers to unity, might not improperly be called the LOGARITHMS of those ratios, since they express the numbers of smaller ratios of which they are composed. And then, for the sake of brevity, they called them the *Logarithms of the said natural numbers themselves*, which are the antecedents of the said ratios to unity, of which they are in truth the representatives.

The foregoing method of considering this property leads to much the same conclusions as the other way, in which the relations between a geometrical series of terms, and their exponents, or the terms of an arithmetical series, are contemplated. In this latter way, it readily occurred that the addition of the terms of the arithmetical series corresponded to the multiplication of the terms of the geometrical series; and that the arithmeticals would therefore form a set of artificial numbers, which, when arranged in tables, with their geometricals, would answer the purposes desired, as has been explained above.

From this property, by assuming four quantities, two of them as two terms in a geometrical series, and the others as the two corresponding terms of the arithmeticals, or artificials, or logarithms, it is evident that all the other terms of both the two series may thence be generated. And therefore there may be as many sets or scales of logarithms as we please, since they depend entirely on the arbitrary assumption of the first two arithmeticals. And all possible natural numbers may be supposed to coincide with some of the terms of any geometrical progression whatever, the logarithms or arithmeticals determining which of the terms in that progression they are.

It was proper however that the arithmetical series should be so assumed, as that the term 0 in it might answer to the term 1 in the geometricals; otherwise the sum of the logarithms of any two numbers would be always to be diminished by the logarithm of 1, to give the logarithm of the product of those numbers: for which reason, making 0 the logarithm of 1, and assuming any quantity whatever for the value of the logarithm of any one number, the logarithms of all other numbers were thence to be derived. And hence, like as the multiplication of two numbers is effected by barely adding their logarithms, so division is performed by subtracting the logarithm of the one from that of the other, raising of powers by multiplying the logarithm of the given number by the index of the power, and extraction of roots by dividing the logarithm by the index of the root. It is also evident that, in all scales or systems of logarithms, the logarithm of 0 will be infinite; namely, infinitely negative if the logarithms increase with the natural numbers, but infinitely positive if the contrary; because that, while the geometrical series must decrease through infinite divisions by the ratio of the progression, before the quotient come to 0 or nothing; the logarithms, or arithmeticals, will in like manner undergo the corresponding infinite subtractions or additions of the common equal difference; which equal increase or decrease, thus indefinitely continued, must needs tend to an infinite result.

This however was no newly-discovered property of numbers, but what was always well known to all mathematicians, being treated of in the writings of Euclid, as also by Archimedes, who made great use of it in his Arenarius, or treatise on the number of the sands, namely, in assigning the rank or place of those terms, of a geometrical series, produced from the multiplication together of any of the foregoing terms, by the addition of the corresponding terms of the arithmetical series, which served as the indices or exponents of the former. Stifelius also treats very fully of this property at folio 35 et seq. and there explains all its principal uses as relating to the logarithms of numbers, only without the name; such as, that addition answers to multiplication, subtraction to division, multiplication of exponents to involution, and dividing of exponents to evolution; all which he exemplifies in the rule-of-three, and in finding several mean proportionals, &c, exactly as is done in logarithms. So that he seems to have been in the full possession of the idea of logarithms, but without the necessity of making a table of such numbers. For the reason why tables of these numbers were not sooner composed, was, that the accuracy and trouble of trigonometrical computations had not sooner rendered them necessary. It is therefore not to be doubted that, about the close of the sixteenth and beginning of the seventeenth century, many persons had thoughts of such a table of numbers, besides the few who are said to have attempted it.

It has been said by some, that Longomontanus invented logarithms: but this cannot well be supposed to have been any more than in idea, since he never published any thing of the kind, nor ever laid claim to the invention, though he lived thirty-three years after they were first published by baron Napier, as he died only in 1647, when they had been long known and received all over Europe. Nay more, Longomontanus himself ascribes the invention to Napier: vid. Astron. Danica, p. 7, &c. Some circumstances of this matter are indeed related by Wood in his "Athenæ Oxonienses," under the article Briggs, on the authority of Oughtred and Wingate, viz. "That one Dr. Craig, a Scotchman, coming out of Denmark into his own country, called upon Joh. Neper baron of Marcheston near Edenburgh, and told him among other discourses, of a new invention in Denmark (by Longomontanus as 'tis said) to save the tedious multiplication and division in astronomical calculations. Neper being solicitous to know farther of him concerning this matter, he could give no other account of it, than that it was by proportionable numbers. Which hint Neper taking, he desired him at his return to call upon him again. Craig, after some weeks had passed, did so, and Neper then showed him a rude draught of that he called *Canon mirabilis Logarithmorum.* Which draught, with some alterations, he printing in 1614, it came forthwith into the hands of our author Briggs, and into those of Will. Oughtred, from whom the relation of this matter came."

Kepler also says, that one Juste Byrge, assistant astronomer to the landgrave of Hesse, invented or projected logarithms long before Neper did; but that they had never come abroad, on account of the great reservedness of their author with regard to his own compositions. It is also said, that Byrge computed a table of natural sines for every two seconds of the quadrant.

But whatever may have been said, or conjectured, concerning any thing that may have been done by others, it is certain that the world is indebted, for the first publication of logarithms, to John Napier, or Nepair*, or in Latin, Neper, baron of Merchiston, or Markinston, in Scotland, who died the 3d of April 1618, at 67 years of age. Baron Napier added considerable improvements to trigonometry, and the frequent numeral computations he performed in this branch, gave occasion to his invention of logarithms, in order to save part of the trouble attending those calculations; and for this reason he adapted his tables peculiarly to trigonometrical uses.

* The origin of which name, Crawfurd informs us, was from a (less) peer*less* action of one of his ancestors, viz. Donald, second son of the earl of Lenox, in the time of David the Second. " Some English writers, mistaking the import of the term *baron*, having called this celebrated person lord Napier, a Scotch nobleman. He was not indeed a peer of Scotland: but the peerage of Scotland informs us, that he was of a very ancient, honourable, and illustrious family; that his ancestors, for many generations, had been possessed of sundry baronies, and, amongst others, of the barony of Merchistoun, which descended to him by the death of his father in 1608. Mr. Briggs, therefore, very properly styles him *Baro Merchestonii.* Now, according to Skene, *de verborum significatione*, ' In this realm (of Scotland) he is called a Baronne, quha haldis his landes immediatelie in chiefe of the king, and hes power of Pit and Gallows; *Fossa et Furca;* quhilk was first institute and granted be king Malcome, quha gave power to the Barrones to have ane Pit, quhairin wemen condemned for thieft suld be drowned, and ane Gallows, whereupon men thieves and trespassowres suld be hanged, conforme to

This discovery he published in 1614, in his book intitled "Mirifici Logarithmorum Canonis Descriptio," reserving the construction of the numbers till the sense of the learned concerning his invention should be known. And, excepting the construction, this is a perfect work on this kind of logarithms, containing in effect the logarithms of all numbers, and the logarithmic sines, tangents, and secants, for every minute of the quadrant, together with the description and uses of the tables, as also his definition and idea of logarithms.

Napier explains his notion of logarithms by lines described or generated by the motion of points, in this manner: He first conceives a line to be generated by the equable motion of a point, which passes over equal portions of it in equal small moments or portions of time: he then considers another line as generated by the unequal motion of a point, in such manner, that, in the aforesaid equal moments or portions of time, there may be described or cut off, from a given line, parts which shall be continually in the same proportion with the respective remainders, of that line, which had before been left: then are the several lengths of the first line, the logarithms of the corresponding parts of the latter. Which description of them is similar to this, that the logarithms are a series of quantities or numbers in arithmetical progression, adapted to another series in geometrical progression. The

the doome given in the Baron Court thereanent.' So that a Scotch baron, though no peer, was nevertheless a very considerable personage, both in dignity and power." *Reid's Essay on Logarithms.*—The name of the illustrious inventor of logarithms, has been variously written at different times, and on different occasions. In his own Latin works, and in (perhaps) all other books in Latin, it is *Neper*, or *Neperus Baro Merchestonii:* By Briggs, in a letter to Archbishop Usher, he is called *Naper, lord of Markinston:* In Wright's translation of the logarithms, which was revised by the author himself, and published in 1616, he is called *Nepair, baron of Merchiston;* and the same by Crawfurd and some others: But M'Kenzie and others write it *Napier, baron of Merchiston;* which, being also the orthography now used by the family, I shall adopt in this work. I observe also, that the Scotch Compendium of Honour says he was only Sir John Napier, and that his son and heir Archibald, was the first lord, being raised to that dignity in 1626. Be this however as it may, I shall conform to the common modes of expression, and call him indifferently, *Baron Napier*, or *Lord Napier*.

first or whole length of the line, which is diminished in geometrical progression, he makes the radius of a circle, and its logarithm 0 or nothing, representing the beginning of the first or arithmetical line; and the several proportional remainders of the geometrical line, are the natural sines of all the other parts of the quadrant, decreasing down to nothing, while the successive increasing values of the arithmetical line, are the corresponding logarithms of those decreasing sines: so that, while the natural lines decrease from radius to nothing, their logarithms increase from nothing to infinite. Napier made the logarithm of radius to be 0, that he might save the trouble of adding or subtracting it, in trigonometrical proportions, in which it so frequently occurred; and he made the logarithms of the sines, from the entire quadrant down to 0, to increase, that they might be positive, and so in his opinion the easier to manage, the sines being of more frequent use than the tangents and secants, of which the whole of the latter and half the former would, in his way, be of a different affection from the sines; for it is evident that the logarithms of all the secants in the quadrant, and of all the tangents above 45°, or the half quadrant, would be negative, being the logarithms of numbers greater than the radius, whose logarithm is made equal to 0 or nothing.

As to the contents of Napier's table; it consists of the natural sines and their logarithms, for every minute of the quadrant. Like most other tables, the arcs are continued to 45 degrees from top to bottom on the left-hand side of the pages, and then returned backwards from bottom to top on the right-hand side of the pages: so that the arcs and their complements, with the sines, natural and logarithmic, stand on the same line of the page, in six columns; and in another column, in the middle of the page, are placed the differences between the logarithmic sines and cosines, on the same lines, and in the adjacent columns on the right and left; thus making in all seven columns in each page. Of these columns, the first and seventh contain the arc and its complement, in degrees and minutes; the second and sixth, the natural sine and cosine of each arc; the third and fifth, the logarithmic sine and cosine; and the fourth, or middle column, the difference between the logarithmic sine and cosine which are in the third and fifth columns. To elucidate the description, the first page of the table is here inserted, as follows.

Gr. min.	0 Sinus.	Logarithmi.	+ | — Differentiæ.	Logarithmi.	Sinus.	
0	0	Infinitum.	Infinitum.	0	10000000	60
1	2909	81425681	81425680	1	10000000	59
2	5818	74494213	74404211	2	9999998	58
3	8727	70439560	70439560	4	9999996	57
4	11636	67562746	67562739	7	9999993	56
5	14544	65331315	65331304	11	9999989	55
6	17453	63508099	63508083	16	9999984	54
7	20362	61966595	61966573	22	9999980	53
8	23271	60631284	60631256	28	9999974	52
9	26180	59453453	59453418	35	9999967	51
10	29088	58399857	58399814	43	9999959	50
11	31997	57446759	57446707	52	9999950	49
12	34906	56576646	56576584	62	9999940	48
13	37815	55776222	55776149	73	9999928	47
14	40724	55035148	55035064	84	9999917	46
15	43632	54345225	54345129	96	9999905	45
16	46541	53699843	53699734	109	9999892	44
17	49450	53093600	53093577	123	9999878	43
18	52359	52522019	52521881	138	9999863	42
19	55268	51981356	51981202	154	9999847	41
20	58177	51468431	51468361	170	9999831	40
21	61086	50980537	50980450	187	9999813	39
22	63995	50515342	50515137	205	9999795	38
23	66904	50070827	50070603	224	9999776	37
24	69813	49645239	49644995	244	9999756	36
25	72721	49237030	49236765	265	9999736	35
26	75630	48844826	48844539	287	9999714	34
27	78539	48467431	48467122	309	9999692	33
28	81448	48103763	48103431	332	9999668	32
29	84357	47752859	47752503	356	9999644	31
30	87265	47413852	47413471	381	9999619	30

Besides the columns which are actually contained in this table, as above exhibited and described, namely, the natural and logarithmic sines, and their differences, the same table is made to serve also for the logarithmic tangents and secants of the whole quadrant, and for the logarithms of common numbers. For, the fourth or middle column contains the logarithmic tangents, being equal to the differences between the logarithmic sines and cosines, when the logarithm of radius is 0, because cosine : sine : : radius : tangent, that is, in logarithms, tangent = sine − cosine. Also the logarithmic sines, made negative, become the logarithmic cosecants, and the logarithmic cosines made negative, are the logarithmic secants; because sine : radius : : radius : cosecant, and cosine : radius : : radius : secant; that is, in logarithms, cosecant = 0 − sine = − sine, and secant = 0 − cosine = − cosine. And to make it answer the purpose of a table of logarithms of common numbers, the author directs to proceed thus: A number being given, find that number in any table of natural sines, or tangents, or secants, and note the degrees and minutes in its arc; then in his table find the corresponding logarithmic sine, or tangent, or secant, to the same number of degrees and minutes; and it will be the required logarithm of the given number.

After his definitions and descriptions of logarithms, Napier explains his table, and illustrates the precepts with examples, showing how to take out the logarithms of sines, tangents, secants, and of common numbers; as also how to add and subtract logarithms. He then proceeds to teach the uses of those numbers; and first, in finding any of the terms of three or four proportionals, showing how to multiply and divide, and to find powers and roots, by logarithms: 2dly, in trigonometry, both plane and spherical, but especially the latter, in which he is very explicit, turning all the theorems for every case into logarithms, computing examples to each in numbers, and then enumerating a set of astronomical problems of the sphere which properly belong to each case. Napier here teaches also some new theorems in spherical trigonometry, particularly, that the tangent of half the base : tang. $\frac{1}{2}$ sum legs :: tang. $\frac{1}{2}$ dif. legs : tang. $\frac{1}{2}$ the alternate base; and the general theorem for what are called his five circular parts, by which he condenses into one rule, in two parts, the theorems for all the cases of right-angled spherical triangles, which had been separately demonstrated by Pitiscus, Lansbergius, Copernicus, Regiomontanus, and others.

The description and use of Napier's canon being in the Latin language, they were translated into English by Mr. Edward Wright, an ingenious mathematician, and inventor of the principles of what has commonly, though erroneously, been called Mercator's Sailing. He sent the translation to the author, at Edinburgh, to be revised by him before publication; who having carefully perused it, returned it with his approbation, and a few lines introduced besides into the translation. But, Mr. Wright dying soon after he received it back, it was after his death published, together with the tables, but each number to one figure less, in the year 1616, by his son Samuel Wright, accompanied with a dedication to the East-India Company, as also a preface by Henry Briggs, of whom we shall presently have occasion to speak more at large, on account of the great share he bore in perfecting the logarithms. In this translation, Mr. Briggs gave also the description and draught of a scale that had been invented by Mr. Wright, and several other methods of his own, for finding the proportional parts to intermediate numbers, the logarithms having been only printed for such numbers as were the natural sines of each minute. And the note which Baron Napier inserted in this English edition, and which was not in the original, was as follows: "But because the addition and subtraction of these former numbers may seem somewhat painful, I intend (if it shall please God) in a second edition, to set out such logarithms as shall make those numbers above written to fall upon decimal numbers, such as 100,000,000, 200,000,000, 300,000,000, &c, which are easie to be added or abated to or from any other number." This note had reference to the alteration of the scale of logarithms, in such manner, that

1 should become the logarithm of the ratio of 10 to 1, instead of the number 2.3025851, which Napier had made that logarithm in his table, and which alteration had before been recommended to him by Briggs, as we shall see presently. Napier also inserted a similar remark in his "Rabdologia," which he printed at Edinburgh in 1617.

The following is the preface to * Wright's book, which, as far as where it mentions the change from the Latin into English, is a literal translation of the preface to Napier's original; but what follows that, is added by Napier himself. And I willingly insert it here, as it contains a declaration of the motives which led to this discovery, and as the book itself is very scarce. "Seeing there is nothing (right well beloved students in the mathematics) that is so troublesome to Mathematicall practise, nor that doth more molest and hinder Calculators, than the Multiplications, Divisions, square and

* Of this ingenious man I shall here insert in a note the following memoirs, as they have been translated from a Latin piece taken out of the annals of Gonvile and Caius College at Cambridge, viz. "This year (1615) died at London, Edward Wright of Garveston in Norfolk, formerly a fellow of this college; a man respected by all for the integrity and simplicity of his manners, and also famous for his skill in the mathematical sciences: insomuch that he was deservedly styled a most excellent mathematician by Richard Hackluyt, the author of an original treatise of our English navigations. What knowledge he had acquired in the science of mechanics, and how usefully he employed that knowledge to the public as well as private advantage, abundantly appear both from the writings he published, and from the many mechanical operations still extant, which are standing monuments of his great industry and ingenuity. He was the first undertaker of that difficult but useful work, by which a little river is brought from the town of Ware in a new canal, to supply the city of London with water; but by the tricks of others he was hindered from completing the work he had begun. He was excellent both in contrivance and execution; nor was he inferior to the most ingenious mechanic in the making of instruments, either of brass, or any other matter. To his invention is owing whatever advantage Hondius's geographical charts have above others; for it was our Wright that taught Jodocus Hondius the method of constructing them, which was till then unknown: but the ungrateful Hondius concealed the name of the true author, and arrogated the glory of the invention to himself. Of this fraudulent practice the good man could not help complaining, and justly enough, in the preface to his Treatise of the Correction of Errors in the Art of Navigation; which he composed with excellent judgment, and after long experience, to the great advancement of naval

cubical Extractions of great numbers, which, besides the tedious expence of time, are for the most part subject to many slippery errors: I began therefore to consider in my minde, by what certaine and ready Art I might remove those hindrances. And having thought upon many things to this purpose, I found at length some excellent briefe rules to be treated of (perhaps) hereafter. But amongst all, none more profitable then this, which together with the hard and tedious Multiplications, Divisions, and Extractions of rootes, doth also cast away from the worke it selfe, even the very numbers themselves that are to be multiplied, divided, and resolved into rootes, and putteth other numbers in their place, which performe as much as they can do, onely by Addition and Subtraction, Division by two, or Division by three; which secret invention, being (as all other good things are) so much the better as it shall be the more common; I thought good

affairs. For the improvement of this art he was appointed mathematical lecturer by the East India Company, and read lectures in the house of that worthy knight Sir Thomas Smith, for which he had a yearly salary of 50 pounds. This office he discharged with great reputation, and much to the satisfaction of his hearers. He published in English, a book on the doctrine of the sphere, and another concerning the construction of sun-dials. He also prefixed an ingenious preface to the learned Gilbert's book on the loadstone. By these and other his writings, he has transmitted his fame to latest posterity. While he was yet a fellow of this college, he could not be concealed in his private study, but was called forth to the public business of the kingdom, by the queen's majesty, about the year 1593. He was ordered to attend the earl of Cumberland in some maritime expeditions. One of these he has given a faithful account of, in the way of a journal or ephemeris, to which he has prefixed an elegant hydrographical chart of his own contrivance. A little before his death, he employed himself about an English translation of the book of logarithms, then lately found out by the honourable Baron Napier, a Scotchman, who had a great affection for him. This posthumous work of his was published soon after, by his only son Samuel Wright, who was also a scholar of this college. He had formed many other useful designs, but was hindered by death from bringing them to perfection. Of him it may be truly said, that he studied more to serve the public than himself; and though he was rich in fame, and in the promises of the great, yet he died poor, to the scandal of an ungrateful age."

Other anecdotes of him, as well as many other mathematical authors, may be found in the curious history of navigation by Dr. James Wilson, prefixed to Mr. Robertson's excellent treatise on that subject.

heretofore to set forth in Latine for the publique use of Mathematicians. But now some of our Countrymen in this Island well affected to these studies, and the more publique good, procured a most learned Mathematician to translate the same into our vulgar English tongue, who after he had finished it, sent the Coppy of it to me, to be seene and considered on by myself. I having most willingly and gladly done the same, finde it to bee most exact and precisely conformable to my minde and the originall. Therefore it may please you who are inclined to these studies, to receive it from me and the Translator, with as much good will as we recommend it unto you. Fare yee well."

There are also extant copies of Wright's translation with the date 1618 in the title: but this is not properly a new edition, being only the old work with a new title-page adapted to it (the old one being cancelled), together with the addition of sixteen pages of new matter, called "An Appendix to the Logarithms, shewing the practice of the calculation of triangles, and also a new and ready way for the exact finding out of such lines and logarithmes as are not precisely to be found in the canons." But we are not told by what author: probably it was by Briggs.

Besides the trouble attending Napier's canon, in finding the proportional parts, when used as a table of the logarithms of common numbers, and which was in part remedied by the fore-mentioned contrivances of Wright and Briggs, it was also accompanied with another inconvenience, which arose from the logarithms being sometimes + or additive, and sometimes − or negative, and which required therefore the knowledge of algebraic addition and subtraction. And this inconvenience was occasioned, partly by making the logarithm of radius to be 0, and the sines to decrease, and partly by the compendious manner in which the author had formed the table; making the three columns of sines, cosines and tangents, to serve also for the other three of cosecants, secants, and cotangents.

But this latter inconvenience was well remedied by John Speidell, in his New Logarithms, first published in 1619, which contained all the six columns, and in this order; sines, cosines, tangents, cotangents, secants, cosecants: and they were besides made all positive, by being taken the arithmetical complements of Napier's, that is, they were the remainders left by subtracting each of these latter from 10000000. And the former inconvenience was more effectually removed by the said Speidell, in an additional table, given in the sixth impression of the former work, in the year 1624. This was a table of Napier's logarithms for the round or integer numbers 1, 2, 3, 4, 5, &c, to 1000, together with their differences and arithmetical complements; as also the halves of the said logarithms, with their differences and arithmetical complements; which halves consequently were the logarithms of the square roots of the said numbers. These logarithms are however a little varied in their form from Napier's, namely, so as to increase *from* 1, whose logarithm is 0, instead of decreasing *to* 1, or radius, whose logarithm was made 0 likewise; that is, Speidell's logarithm of any number n, is equal to Napier's logarithm of its reciprocal $\frac{1}{n}$: so that in this last table of Speidell's, the logarithm of 1 being 0, the logarithm of 10 is 2302584, the logarithm of 100 is twice as much, or 4605168, and that of 1000 thrice as much, or 6907753.

This table is now commonly called *hyperbolic* logarithms, because the numbers express the areas between the asymptote and curve of the hyperbola, those areas being limited by ordinates parallel to the other asymptote, and the ordinates decreasing in geometrical progression. But this is not a very proper method of denominating them, as such areas may be made to denote any system of logarithms whatever, as will be shown more at large in the proper place.

In the year 1619, Robert Napier, son of the inventor of logarithms, published a new edition of his late father's "Logarithmorum Canonis Descriptio," together with the promised "Logarithmorum Canonis Constructio," and other miscellaneous pieces, written by his father and Mr. Briggs.—Also one Bartholomew Vincent, a bookseller at Lugdunum, or Lyons, in France, printed there an exact copy of the same two works in one volume, in the year 1620; which was four years before the logarithms were carried to France by Wingate, who was therefore erroneously said to have first introduced them into that country. But we shall treat more particularly of the contents of this work, after having enumerated the other writers on this sort of logarithms.

In 1618 or 1619, Benjamin Ursinus, mathematician to the Elector of Brandenburg, published, at Cologn, his "Cursus Mathematicus," in which is contained a copy of Napier's logarithms, with the addition of some tables of proportional parts. And in 1624, he printed at the same place, his "Trigonometria," with a table of natural sines and their logarithms, of the Napierian kind and form, to every ten seconds in the quadrant; which he had been at much pains in computing.

In the same year 1624, logarithms, of nearly the same kind, were also published, at Marpurg, by the celebrated John Kepler, mathematician to the Emperor Ferdinand the Second, under the title of "Chilias Logarithmorum ad Totidem Numeros Rotundos, præmissa Demonstratione legitima Ortus Logarithmorum eorumque Usus," &c; and the year following, a supplement to the same; being applied to round or integer numbers, and to such natural sines as nearly coincide with them. These are exactly the same kind of logarithms as Napier's, being the same logarithms of the natural sines of arcs, beginning from the quadrant, whose sine or radius is 10,000,000, the logarithm of which is made 0, and from thence the sines decreasing by equal differences, down to 0, or the beginning of the quadrant, while their logarithms increase to infinity. So that the difference between this table and Napier's, consists only in this, namely, that in Napier's table the *arc* of the quadrant is divided into equal parts, differing by one minute each, and consequently their sines, to which the logarithms are adapted, are irrational or interminate numbers, and only expressed by approximate decimals; whereas in Kepler's table, the *radius* is divided into equal parts, which are considered as perfect and terminate sines, having equal differences, and to which terminate sines the logarithms are here adapted. By this means indeed the proportions for intermediate numbers and logarithms are easier made; but then the corresponding arcs are not terminate, being irrational, and only set down to an approximate degree. So that Kepler's table is more convenient as a table of the logarithms of common numbers, and Napier's as the logarithmic sines of the arcs of the quadrant. In both tables, the logarithm of the ratio of 10 to 1, is the same quantity, namely 23025852; and as the radius, or greatest sine, is 10,000,000, whose logarithm is made 0, the logarithms of the decuple parts of it will be found by adding 23025852 continually, or multiplying this logarithm by 2, 3, 4, &c; and hence the logarithm of 1, the first number, or smallest sine, in the table, is 161180959, or 7 times 2302 &c.

Besides the two columns, of the natural sines and their logarithms, with the differences of the logarithms, this table of Kepler's consists also of three other columns; the first of which contains the nearest arcs, belonging to those sines, expressed in degrees, minutes and seconds; and the other two express what parts of the radius each sine is equal to, namely, the one of them in 24th parts of the radius, and minutes and seconds of them; and the other in 60th parts of the radius, and minutes of them. The following specimen is extracted from the last page of the table, printed exactly as in the work itself.

Arcus Circuli cum differentiis.	Sinus seu numeri absoluti.	Partes vicesimæ quartæ.	Logarithmi cum differentiis.	Partes sexagenariæ.
19. 34			101.58	
80. 3. 46	98500.00	23. 38. 24	1511.36+	59. 6
20. 12			101.47	
80. 23. 58	98600.00	23. 39. 50	1409.89+	59. 10
20. 53			101.37	
80. 44. 51	98700.00	23. 41. 17	1308.52+	59. 13
21. 42			101.26	
81. 6. 33	98800.00	23. 42. 43	1207.26	59. 17
22. 53			101.17	
81. 29. 26	98900.00	23. 44. 10	1106.09+	59. 20
24. 6			101.06	
81. 53. 32	99000.00	23. 45. 36	1005.03+	59. 24
25. 6			100.96	
82. 18. 38	99100.00	23. 47. 2	904.07+	59. 28
26. 28			100.85	
82. 45. 6	99200.00	23. 48. 29	803.22+	59. 31
27. 54			100.76	
83. 13. 0	99300.00	23. 49. 55	702.46	59. 35
30. 20			100.65	
83. 43. 20	99400.00	23. 51. 22	601.81	59. 38
32. 40			100.56	
84. 16. 0	99500.00	23. 52. 48	501.25+	59. 42
36. 30			100.45	
84. 52. 30	99600.00	23. 54. 14	400.80	59. 46
41. 9			100.35	
85. 33. 39	99700.00	23. 55. 41	300.45	59. 49
48. 54			100.25	
86. 22. 33	99800.00	23. 57. 7	200.20	59. 53
1. 3. 42			100.15	
87. 26. 15	99900.00	23. 58. 34	100.05	59. 56
2. 33. 45			100.05	
90. 0. 0	100000.00	24. 0. 0	000000.00	60. 0

To the table, Kepler prefixes a pretty considerable tract, containing the construction of the logarithms, and a demonstration of their properties and structure, in which he considers logarithms, in the true and legitimate way, as the measures of ratios, as shall be shown more particularly hereafter in the next tract, where the construction of logarithms is fully treated on.

Kepler also introduced the logarithmic calculus into his Rudolphine tables, published in 1627; and inserted in that work several logarithmic tables; as, first a table similar to that above described, except that the second, or column of sines, or of absolute numbers, is omitted, and, instead of it, another column is added, showing what part of the quadrant each arc is equal to, namely the quotient, expressed in integers and sexagesimal parts, arising from dividing the whole quadrant by each given arc; 2dly, Napier's table of logarithmic sines, to every minute of the quadrant; also two other smaller tables, adapted to the purposes of eclipses and the latitudes of the planets. In this work also, Kepler gives a succinct account of logarithms, with the description and use of those that are contained in these tables. And here it is that he mentions Justus Byrgius, as having had logarithms before Napier published them.

Besides the above, some few others published logarithms of the same kind, about this time. But let us now return to treat of the history of the common or Briggs's logarithms, so called because he first computed them, and first mentioned them, and recommended them to Napier, instead of the first kind by him invented.

Mr. Henry Briggs, not less esteemed for his great probity, and other eminent virtues, than for his excellent skill in mathematics, was, at the time of the publication of Napier's logarithms, in 1614, professor of geometry in Gresham college in London, having been appointed the first professor after its institution: which appointment he held till January 1620, when he was chosen, also the first, Savilian professor of geometry at Oxford, where he died January the 26th, $163\frac{0}{1}$, aged about 74 years.

On the publication of Napier's logarithms, Briggs immediately applied himself to the study and improvement of them. In a letter to Mr. (afterwards Archbishop) Usher, dated the

10th of March 1615, he writes, "that he was wholly taken up and employed about the noble invention of logarithms, lately discovered." And again, "Napier lord of Markinston hath set my head and hands at work with his new and admirable logarithms: I hope to see him this summer, if it please God; for I never saw a book which pleased me better, and made me more wonder." Thus we find that Briggs began very early to compute logarithms: but these were not of the same kind with Napier's, in which the logarithm of the ratio of 10 to 1 was 2.3025851 &c; for, in Briggs's first attempt he made 1 the logarithm of that ratio; and, from the evidence we have, it appears that he was the first person who formed the idea of this change in the scale, which he presently and liberally communicated, both to the public in his lectures, and to lord Napier himself, who afterwards said that he also had thought of the same thing; as appears by the following extract, translated from the preface to Briggs's "Arithmetica Logarithmica:" "Wonder not (says he) that these logarithms are different from those which the excellent baron of *Marchiston* published in his Admirable Canon. For when I explained the doctrine of them to my auditors at Gresham college in London, I remarked that it would be much more convenient, the logarithm of the sine total or radius being 0 (as in the *Canon Mirificus*), if the logarithm of the 10th part of the said radius, namely, of 5° 44′ 21″, were 100000 &c; and concerning this I presently wrote to the author; also, as soon as the season of the year and my public teaching would permit, I went to Edinburgh, where being kindly received by him, I staid a whole month. But when we began to converse about the alteration of them, he said that he had formerly thought of it, and wished it; but that he chose to publish those that were already done, till such time as his leisure and health would permit him to make others more convenient. And as to the manner of the change, he thought it more expedient that 0 should be made the logarithm of 1, and 100000 &c the logarithm of radius; which I could not but acknowledge was much better. Therefore, rejecting those which I had before prepared, I proceeded, at his exhortation, to calculate these: and the next summer I went again to Edinburgh, to shew him the principle of them; and should have been glad to do the same the third summer, if it had pleased God to spare him so long."

So that it is plain that Briggs was the inventor of the present scale of logarithms, in which 1 is the logarithm of the ratio of 10 to 1, and 2 that of 100 to 1, &c; and that the share which Napier had in them, was only advising Briggs to begin at the lowest number 1, and make the logarithms, or artificial numbers, as Napier had also called them, to *increase* with the natural numbers, instead of *decreasing*; which made no alteration in the figures that expressed Briggs's logarithms, but only in their affection or signs, changing them from negative to positive; so that Briggs's first logarithms to the numbers in the second column of the annexed tablet, would have been as in the first column; but after they were changed, as they are here in the third column; which is a change of no essential difference, as the logarithm of the ratio of 10 to 1, the radix of the natural system of numbers, continues the same; and a change in the logarithm of that ratio being the only circumstance that can essentially alter the system of logarithms, the logarithm of 1 being 0. And the reason why Briggs, after that interview, rejected what he had before done, and began anew, was probably because he had adapted his new logarithms to the approximate sines of arcs, instead of to the round or integer numbers; and not from their being logarithms of another system, as were those of Napier.

B	Num.	N
n	$\cdot01^n$	$-n$
3	·001	−3
2	·01	−2
1	·:	−1
0	1	0
−1	10	1
−2	100	2
−3	1000	3
$-n$	10^n	n

On Briggs's return from Edinburgh to London the second time, namely, in 1617, he printed the first thousand logarithms, to eight places of figures, besides the index, under the title of "Logarithmorum Chilias Prima." Though these seem not to have been published till after death of Napier, which happened on the 3d of April 1618, as before said; for, in the preface to them, Briggs says, "Why these logarithms differ from those set forth by their most illustrious inventor, of ever respectful memory, in his 'Canon Mirificus,' IT IS TO BE HOPED his posthumous work will shortly make appear." And as Napier, after communication had with Briggs on the subject of altering the scale of logarithms, had given notice, both in Wright's translation, and in his own "Rabdologia," printed in 1617, of his intention to alter the scale, (though it appears very plainly that he never intended to compute any more), without making any mention of the share which Briggs had in the alteration, this gentleman modestly gave the above hint. But not finding any regard paid to it in the said posthumous work, published by lord Napier's son in 1619, where the alteration is again adverted to, but still without any mention of Briggs; this gentleman thought he could not do less than state the grounds of that alteration himself, as they are above extracted from his work published in 1624.

Thus, upon the whole matter, it seems evident that Briggs, whether he had thought of this improvement in the construction of logarithms, of making 1 the logarithm of the ratio of 10 to 1, before lord Napier, or not (which is a secret that could be known only to Napier himself), was the first person who communicated the idea of such an improvement to the world; and that he did this in his lectures to his auditors at Gresham college in the year 1615, very soon after his perusal of Napier's "Canon Mirificus Logarithmorum," published in the year 1614. He also mentioned it to Napier, both by letter in the same year, and on his first visit to him in Scotland in the summer of the year 1616, when Napier approved the idea, and said it had already occurred to himself, and that he had determined to adopt it. It appears therefore, that it would have been more candid in lord Napier to have told the world, in the second edition of this book, that Mr. Briggs had mentioned this improvement to him, and that he had thereby been confirmed in the resolution he had already taken, before

Mr. Briggs's communication with him (if indeed that was the fact), to adopt it in that his second edition, as being better fitted to the decimal notation of arithmetic which was in general use. Such a declaration would have been but an act of justice to Mr. Briggs; and the not having made it, cannot but incline us to suspect that lord Napier was desirous that the world should ascribe to him alone the merit of this very useful improvement of the logarithms, as well as that of having originally invented them; though, if the having first communicated an invention to the world be sufficient to entitle a man to the honour of having first invented it, Mr. Briggs had the better title to be called the first inventor of this happy improvement of logarithms.

In 1620, two years after the "Chilias Prima" of Briggs came out, Mr. Edmund Gunter published his "Canon of Triangles," which contains the artificial or logarithmic sines and tangents, for every minute, to seven places of figures, besides the index, the logarithm of radius being 10·0 &c. These logarithms are of the kind last agreed upon by Napier and Briggs, and they were the first tables of logarithmic sines and tangents that were published of this sort. Gunter also, in 1623, reprinted the same in his book "De Sectore et Radiâ," together with the "Chilias Prima" of his old colleague Mr. Briggs, he being professor of astronomy at Gresham college when Briggs was professor of geometry there, Gunter having been elected to that office the 6th of March 1619, and enjoyed it till his death, which happened on the 10th of December 1626, about the forty-fifth year of his age. In 1623, also, Gunter applied these logarithms of numbers, sines, and tangents, to straight lines drawn on a ruler; with which, proportions in common numbers and trigonometry were resolved by the mere application of a pair of compasses; a method founded on this property, that the logarithms of the terms of equal ratios are equidifferent. This instrument, in the form of a two-foot scale, is now in common use for navigation and other purposes, and is commonly called the Gunter. He also greatly improved the sector for the same uses. Gunter was the first who used the word *cosine* for the sine of the complement of an arc. He also introduced the use of arithmetical complements into the logarithmical arithmetic, as is witnessed by Briggs, chap. 15, Arith. Log. And it has been said, that he started the idea of the logarithmic curve, which was so called because the segments of its axis are the logarithms of the corresponding ordinates.

The logarithmic lines were afterwards drawn in various other ways. In 1627, they were drawn by Wingate on two separate rulers sliding against each other, to save the use of compasses in resolving proportions. They were also, in 1627, applied to concentric circles, by Oughtred. Then in a spiral form, by a Mr. Milburne of Yorkshire, about the year 1650. And, lastly, in 1657, on the present sliding rule, by Seth Partridge.

The discoveries relating to logarithms were carried to France by Mr. Edmund Wingate, but not first of all, as he erroneously says in the preface to his book. He published at Paris, in 1624, two small tracts in the French language; and afterwards at London, in 1626, an English edition of the same, with improvements. In the first of these, he teaches the use of Gunter's rules; and in the other, that of Briggs's logarithms, and the artificial sines and tangents. Here are contained, also, tables of those logarithms, sines, and tangents, copied from Gunter. The edition of these logarithms printed at London in 1635, and the former editions also I suppose, has the units figures disposed along the tops of the columns, and the tens down the margins, like our tables at present; with the whole logarithm, which was only to fix places of figures, in the angle of meeting: which is the first instance that I have seen of this mode of arrangement.

But proceed we now to the larger structure of logarithms. Briggs had continued from the beginning to labour with great industry at the computation of those logarithms of which he before published a short specimen in small numbers. And, in 1624, he produced his " Arithmetica Logarithmica"—a stupendous work for so short a time !—containing the logarithms of 30000 natural numbers, to fourteen places of figures besides the index, namely, from 1 to 20000, and from 90000 to 100000; together with the differences of the logarithms. Some writers say that there was another *chiliad*, namely, from 100000 to 101000; but none of the copies that I have seen have more than the 30000 above mentioned, and they were all regularly terminated in the usual way with the word FINIS. The preface to these logarithms contains, among other things, an account of the alteration made in the scale by Napier and himself, from which we have given an extract; and an earnest solicitation to others to undertake the computation for the intermediate numbers, offering to give instructions, and paper ready ruled for that purpose, to any persons so inclined to contribute to the completion of so valuable a work. In the introduction, he gives also an ample treatise on the construction and uses of these logarithms, which will be particularly described hereafter.—By this invitation, and other means, he had hopes of collecting materials for the logarithms of the intermediate 70000 numbers, while he should employ his own labour more immediately on the canon of logarithmic sines and tangents, and so carry on both works at once; as indeed they were both equally necessary, and he himself was now pretty far advanced in years.

Soon after this however, Adrian Vlacq, or Flack, of Gouda in Holland, completed the intermediate seventy chiliads, and republished the "Arithmetica Logarithmica" at that place, in 1627 and 1628, with those intermediate numbers, making in the whole the logarithms of all numbers to 100000, but only to ten places of figures. To these was added a table of artificial sines, tangents, and secants, to every minute of the quadrant.

Briggs himself lived also to complete a table of logarithmic sines and tangents for the hundredth part of every degree, to fourteen places of figures besides the index; together with a table of natural sines for the same parts to fifteen places, and the tangents and secants for the same to ten places; with the construction of the whole. These tables were printed at

Gouda, under the care of Adrian Vlacq, and mostly finished off before 1631, though not published till 1633. But his death, which then happened, prevented him from completing the application and uses of them. However, the performing of this office, when dying, he recommended to his friend Henry Gellibrand, who was then professor of astronomy in Gresham college, having succeeded Mr. Gunter in that appointment. Gellibrand accordingly added a preface, and the application of the logarithms to plain and spherical trigonometry, &c; and the whole was printed at Gouda by the same printer, and brought out in the same year, 1633, as the "Trigonometria Artificialis" of Vlacq, who had the care of the press as above said. This work was called "Trigonometria Britannica;" and besides the arcs in degrees and centesms of degrees, it has another column, containing the minutes and seconds answering to the several centesms in the first column.

In 1633, as mentioned above, Vlacq printed at Gouda, in Holland, his "Trigonometria Artificialis; sive Magnus Canon Triangulorum Logarithmicus ad Decadas Secundorum Scrupulorum constructus." This work contains the logarithmic sines and tangents to ten places of figures, with their differences, for every ten seconds in the quadrant. To them is also added Briggs's table of the first 20000 logarithms, but carried only to ten places of figures besides the index, with their differences. The whole is preceded by a description of the tables, and the application of them to plane and spherical trigonometry, chiefly extracted from Briggs's "Trigonometria Britannica," mentioned above.

Gellibrand published also, in 1635, "An Institution Trigonometricall," containing the logarithms of the first 10000 numbers, with the natural sines, tangents, and secants, and the logarithmic sines and tangents, for degrees and minutes, all to seven places of figures, besides the index; as also other tables proper for navigation; with the uses of the whole. Gellibrand died the 9th of February 1636, in the 40th year of his age, to the great loss of the mathematical world.

Besides the persons hitherto mentioned, who were mostly computers of logarithms, many others have also published tables of those artificial numbers, more or less complete, and sometimes improved and varied in the manner and form of them. We may here just advert to a few of the principal of these.

In 1626, D. Henrion published, at Paris, a treatise concerning Brigg's logarithms of common numbers, from 1 to 20000, to eleven places of figures; with the sines and tangents to eight places only.

In 1631, was printed, at London, by one George Miller, a book containing Briggs's logarithms, with their differences, to ten places of figures besides the index, for all numbers to 100000; as also the logarithmic sines, tangents, and secants, for every minute of the quadrant; with the explanation and uses in English.

The same year, 1631, Richard Norwood published his "Trigonometria;" in which we find Briggs's logarithms for all numbers to 10000, and for the sines, tangents, and secants, to every minute, both to seven places besides the index.—In the conclusion of the trigonometry, he complains of the unfair practices of printing Vlacq's book in 1627 or 1628, and the book mentioned in the last article. His words are, "Now, whereas I have here, and in sundry places in this book, cited Mr. Briggs his 'Arithmetica Logarithmica,' (lest I may seem to abuse the reader) you are to understand not the book put forth about a month since in English, as a translation of his, and with the same title; being nothing like his, nor worthy his name; but the book which himself put forth with this title in Latin, being printed at London anno 1624. And here I have just occasion to blame the ill dealing of these men, both in the matter before mentioned, and in printing a second edition of his 'Arithmetica Logarithmica' in Latin, whilst he lived, against his mind and liking; and brought them over to sell, when the first were unsold; so frustrating those additions which Mr. Briggs intended in his second edition, and moreover leaving out some things that were in the first edition, of special moment: a practice of very ill consequence, and tending to the great disparagement of such as take pains in this kind."

Francis Bonaventure Cavalerius published at Bologna, in 1632, his "Directorium Generale Uranometricum," in which are tables of Briggs's logarithms of sines, tangents, secants, and versed sines, each to eight places, for every second of the first five minutes, for every five seconds from five to ten minutes, for every ten seconds from ten to twenty minutes, for every twenty seconds from twenty to thirty minutes, for every thirty seconds from 30′ to 1° 30′, and for every minute in the rest of the quadrant; which is the first table of logarithmic versed sines that I know of. In this book are contained also the logarithms of the first ten chiliads of natural numbers, namely, from 1 to 10000, disposed in this manner: all the twenties at top, and from 1 to 19 on the side, the logarithm of the sum being in the square of meeting. In this work also, I think Cavalerius gave the method of finding the area or spherical surface contained by various arcs described on the surface of a sphere; which had before been given by Albert Girard, in his Algebra, printed in the year 1629.

Also, in the "Trigonometria" of the same author, Cavalerius, printed in 1643, besides the logarithms of numbers from 1 to 1000, to eight places, with their differences, we find both natural and logarithmic sines, tangents, and secants, the former to seven, and the latter to eight places; namely, to every 10″ of the first 30 minutes, to every 30″ from 30′ to 1°; and the same for their complements, or backwards through the last degree of the quadrant; the intermediate 88° being to every minute only.

Mr. Nathaniel Roe, "Pastor of Benacre in Suffolke," also reduced the logarithmic tables to a contracted form, in his "Tabulæ Logarithmicæ," printed at London in 1633. Here we have Briggs's logarithms of numbers from 1 to 100000, to eight places; the fifties placed at top, and from 1 to 50 on the side; also the first four figures of the logarithms at top, and the other four down the columns. They contain also the logarithmic sines and tangents to every 100th part of degrees, to ten places.

Ludovicus Frobenius published at Hamburgh, in 1634, his "Clavis Universa Trigonometriæ," containing tables of Briggs's logarithms of numbers, from 1 to 2000; and of sines, tangents, and secants, for every minute; both to seven places.

But the table of logarithms of common numbers was reduced to its most convenient form by John Newton, in his "Trigonometria Britannica," printed at London in 1658, having availed himself of both the improvements of Wingate and Roe, namely, uniting Wingate's disposition of the natural numbers with Roe's contracted arrangement of the logarithms, the numbers being all disposed as in our best tables at present, namely, the units along the top of the page, and the tens down the left-hand side, also the first three figures of each logarithm in the first column, and the remaining five figures in the other columns, the logarithms being to eight places. This work contains also the logarithmic sines and tangents, to eight figures besides the index, for every 100th part of a degree, with their differences, and for 1000th parts in the first three degrees.—In the preface to this work, Newton takes occasion, as Wingate and Norwood had done before, as well as Briggs himself, to censure the unfair practices of some other publishers of logarithms. He says, "In the second part of this institution, thou art presented with Mr. Gellibrand's Trigonometrie, faithfully translated from the Latin copy, that which the author himself published under the title of 'Trigonometria Britannica,' and not that which Vlacq the Dutchman styles 'Trigonometria Artificialis,' from whose corrupt and imperfect copy that seems to be translated which is amongst us generally known by the name of 'Gellibrand's Trigonometry;' but those who either knew him, or have perused his writings, can testify that he was no admirer of the old sexagenary way of working; nay, that he did preferre the decimal way before it, as he hath abundantly testified in all the examples of this his Trigonometry, which differs from that other which Vlacq hath published, and that which hath hitherto borne his name in English, as in the form, so likewise in the matter of it; for in the two last-mentioned editions, there is something left out in the second chapter of plain triangles, the third chapter wholly omitted, and a part of the third in the spherical; but in this edition nothing: something we have added to both, by way of explanation and demonstration."

In 1670, John Caramuel published his "Mathesis Nova," in which are contained 1000 logarithms both of Napier's and Briggs's form, as also 1000 of what he calls the Perfect Logarithms, namely, the same as those which Briggs first thought of, which differ from the last only in this, that the one increases while the other decreases, the radix or logarithm of the ratio of 10 to 1 being the same in both.

The books of logarithms have since become very numerous, but the logarithms are mostly of that sort invented by Briggs, and which are now in common use. Of these, the most noted for their accuracy or usefulness, besides the works above mentioned, are Vlacq's small volume of tables, particularly that edition printed at Lyons, in 1670; also tables printed at the same place in 1760; but most especially the tables of Sherwin and Gardiner, particularly my own improved editions of them. Of these, Sherwin's "Mathematical Tables," in 8vo, formed, till lately, the most complete collection of any, containing, besides the logarithms of all numbers to 101000, the sines, tangents, secants, and versed sines, both natural and logarithmic, to every minute of the quadrant, though not conveniently arranged. The first edition was in 1706; but the third edition, in 1742, which was revised by Gardiner, is esteemed the most correct of any, though containing many thousands of errors in the final figures, as well as all the former editions: as to the last or fifth edition, in 1771, it is so erroneously printed that no dependance can be placed in it, being the most inaccurate book of tables I ever knew; I have a list of several thousand errors which I have corrected in it, as well as in Gardiner's octavo edition, and in Sherwin's edition.

Gardiner also printed at London, in 1742, a quarto volume of "Tables of Logarithms, for all numbers from 1 to 102100, and for the sines and tangents to every ten seconds of each degree in the quadrant; as also, for the sines of the first 72 minutes to every single second: with other useful and necessary tables;" namely a table of Logistical Logarithms, and three smaller tables to be used for finding the logarithms of numbers to twenty places of figures. Of these tables of Gardiner, only a small number was printed, and that by subscription; and they have always been held in great estimation for their accuracy and usefulness.

An edition of Gardiner's collection was also elegantly printed at Avignon in France, in 1770, with some additions, namely, the sines and tangents for every single second in the first four degrees, and a small table of hyperbolic logarithms, copied from a treatise on Fluxions by the late ingenious Mr. Thomas Simpson: but this is not quite so correct as Gardiner's own edition. The tables in all these books are to seven places of figures.

Lastly, my own Mathematical Tables, being the most accurate and best arranged set of logarithmic tables ever before given; preceded also by a large and critical history of Trigonometry and Logarithms, and terminating with a copious list of the errors discovered in the principal other tables of this kind.

There have also lately appeared the following accurate and elegant books of logarithms; viz. 1. "Logarithmic Tables," by the late Mr. Michael Taylor, a pupil of mine, and author of "The Sexagesimal Table." His work consists of three tables; 1st, The Logarithms of Common Numbers from 1 to 1260, each to 8 places of figures; 2dly, The Logarithms of all Numbers from 1 to 101000, each to 7 places; 3dly, The Logarithmic Sines and Tangents to every Second of the Quadrant, also to 7 places of figures: a work that must prove highly useful to such persons as may be employed in very nice and accurate calculations, such as astronomical tables, &c. The author dying when the tables were nearly all printed off, the Rev. Dr. Maskelyne, astronomer royal, supplied a preface, containing an account of the work, with excellent precepts for the explanation and use of the tables: the whole very accurately and elegantly printed on large 4to, 1792.

2. "Tables Portatives de Logarithmes, publiées à Londres par Gardiner," &c. This work is most beautifully printed in a neat portable 8vo volume, and contains all the tables in Gardiner's 4to volume, with some additions and improvements, and with a considerable degree of accuracy. Printed at Paris, by Didot, 1793. On this, as well as several other occasions, it is but justice to remark the extraordinary spirit and elegance with which the learned men, and the artisans of the French nation, undertake and execute works of merit.

3. A second edition of the "Tables Portatives de Logarithmes," &c. printed at Paris with the stereotypes, of solid pages, in 8vo, 1795, by Didot. This edition is greatly enlarged, by an extension of the old tables, and many new ones; among which are the logarithm sines and tangents to every ten thousandth part of the quadrant, viz. in which the quadrant is first divided into 100 equal parts, and each of these into 100 parts again.

4. Other more extensive tables, by Borda and Delambre, were published at Paris in 1801. Besides the usual table of the logarithms of common numbers, and a large introduction, on the nature and construction of them, this work contains very extensive tables of decimal trigonometry, arranged in a new and curious way, and containing the log. sines, tangents, and secants, of the quadrant, divided first into 100 degrees, each degree into 100 minutes, and each minute into 100 seconds.

The logarithmic canon serves to find readily the logarithm of any assigned number; and we are told by Dr. Wallis, in the second volume of his Mathematical Works, that an antilogarithmic canon, or one to find as readily the number corresponding to every logarithm, was begun, he thinks, by Harriot the algebraist, who died in 1621, and completed by Walter Warner, the editor of Harriot's works, before 1640;

which ingenious performance, it seems, was lost, for want of encouragement to publish it.

A small specimen of such numbers was published in the Philosophical Transactions for the year 1714, by Mr. Long of Oxford; but it was not till 1742 that a complete antilogarithmic canon was published by Mr. James Dodson, wherein he has computed the numbers corresponding to every logarithm from 1 to 100000, for 11 places of figures.

TRACT XXI.

THE CONSTRUCTION OF LOGARITHMS, &c.

Having, in the last Tract, described the several kinds of logarithms, their rise and invention, their nature and properties, and given some account of the principal early cultivators of them, with the chief collections that have been published of such tables; proceed we now to deliver a more particular account of the ideas and methods employed by each author, and the peculiar modes of construction made use of by them. And first, of the great inventor himself, Lord Napier.

Napier's Construction of Logarithms.

The inventor of logarithms did not adapt them to the series of natural numbers 1, 2, 3, 4, 5, &c, as it was not his principal idea to extend them to all arithmetical operations in general; but he confined his labours to that circumstance which first suggested the necessity of the invention, and adapted his logarithms to the approximate numbers which express the natural sines of every minute in the quadrant, as they had been set down by former writers on trigonometry.

The same restricted idea was pursued through his method of constructing the logarithms. As the lines of the sines of all arcs are parts of the radius, or sine of the quadrant, which