TRACT XIX.

HISTORY OF TRIGONOMETRICAL TABLES, &c.

Necessity, the fruitful mother of most useful inventions; gave birth to the various numeral tables employed in trigonometry, astronomy, navigation, &c. Astronomy has been cultivated from the earliest ages. The progress of that science, requiring numerous arithmetical computations of the sides and angles of triangles, both plain and spherical, gave rise to trigonometry; for those frequent calculations suggested the necessity of performing them by the property of similar triangles; and for the ready application of this property, it was necessary that certain lines described in and about circles, to a determinate radius, should be computed, and disposed in tables. Navigation, and the continually improving accuracy of astronomy, have also occasioned as continual an increase in the accuracy and extent of those tables. And this, it is evident, must ever be the case, the improvement of trigonometry uniformly following the improvement of those other useful sciences, for the sake of which it is more especially cultivated.

The ancients performed their trigonometry by means of the chords of arcs, which, with the chords of their supplemental arcs, and the constant diameter, formed all species of right-angled triangles. Beginning with the radius, and the arc whose chord is equal to the radius, they divided them both into 60 equal parts, and estimated all other arcs and chords by those parts, namely all arcs by 60ths of that arc, and all chords by 60ths of its chord or of the radius. At least this method is as old as the writings of Ptolemy, who used the sexagenary arithmetic for this division of chords and arcs, and for astronomical purposes.—And this, by-the-bye, may be the reason why the whole circumference is divided into 360, or 6 times 60, equal parts or degrees, the whole circumference being equal to 6 times the first arc, whose chord is equal to the radius: unless perhaps we are rather to seek for the division of the circle in the number of days in the year; for thus, the ancient year consisting of 360 days, the sun or earth in each day described the 360th part of the orbit; and thence might arise the method of dividing every circle into 360 parts; and, radius being equal to the chord of 60 of those parts, the sexagesimal division, both of the radius and of the parts, might thence follow. Trigonometry however must have been cultivated long before the time of Ptolemy; and indeed Theon, in his commentary on Ptolemy's Almagest, l. i. ch. 9, mentions a work of the philosopher Hipparchus, written about a century and a half before Christ, consisting of 12 books on the chords of circular arcs; which must have been a treatise on trigonometry. And Menelaus also, in the first century of Christ, wrote 6 books concerning subtenses or chords of arcs. He used the word *nadir*, of an arc, which he defined to be the right line subtending the double of the arc; so that his nadir of an arc was the double of our sine of the same arc, or the chord of the double arc; and therefore whatever he proves of the former, may be applied to the latter, substituting the double sine for the nadir.

The radius has since been decimally divided; but the sexagesimal divisions of the arc have continued in use to this day. Indeed our countrymen Briggs and Gellibrand, having a general dislike to all sexagesimal divisions, made an attempt at some reformation of this custom, by dividing the degrees of the arcs, in their tables, into centesms or hundredth parts, instead of minutes or 60th parts. The same was also recommended by Vieta and others; and a decimal division of the whole quadrant might perhaps soon have followed, had it not been for the tables of Vlacq, which came out a little after, to every 10 seconds, or 6th parts of minutes.—But the complete reformation would be, to express all arcs by their real lengths, namely in equal parts of the radius decimally divided: according to which method I have nearly completed a table of sines and tangents.

It is not to be doubted that many of the ancients wrote on the subject of trigonometry, being a necessary part of astronomy; though few of their labours on that branch have come to our knowledge, and still fewer of the writings themselves have been handed down to us.

We are in possession of the three books of Menelaus, on spherical trigonometry; but the six books are lost which he wrote upon chords, being probably a treatise on the construction of trigonometrical tables.

The trigonometry of Menelaus was much improved by Ptolemy (Claudius Ptolemæus) the celebrated philosopher and mathematician. He was born at Pelusium; taught astronomy at Alexandria in Egypt; and died in the year of Christ 147, being the 78th year of his age. In the first book of his Almagest, Ptolemy delivers a table of arcs and chords, with the method of construction. This table contains 3 columns; in the 1st are the arcs to every half degree or 30 minutes; in the 2d are their chords, expressed in degrees, minutes and seconds; of which degrees the radius contains 60; and in the 3d column are the differences of the chords answering to 1 minute of the arcs, or the 30th part of the differences between the chords in the 2d column. In the construction of this table, among other theorems, Ptolemy shows, for the first time that we know of, this property of any quadrilateral inscribed in a circle, namely, that the rectangle under the two diagonals, is equal to the sum of the two rectangles under the opposite sides.

This method of computation, by the chords, continued in use till about the middle centuries after Christ; when it was changed for that of the sines, which were about that time introduced into trigonometry by the Arabians, who in other respects much improved this science, which they had received from the Greeks, introducing, among other things, the three or four theorems, or axioms, which we make use of at present, as the foundation of our modern trigonometry.

The other great improvements, that have been made in this branch, are due to the Europeans. These improvements they have gradually introduced since they received this science from the Arabians. And though these latter people had long used the Indian or decimal scale of arithmetic, it does not appear that they varied from the Greek or sexagesimal division of the radius, by which the chords and sines had been expressed.

This alteration, it is said, was first made by George Purbach, who was so called from his being a native of a place of that name, between Austria and Bavaria. He was born in 1423, and studied mathematics and astronomy at the university of Vienna, where he was afterwards professor of those sciences, though but for a short time, the learned world quickly suffering a great loss by his immature death, which happened in 1462, at the age of 39 years only. Purbach, besides enriching trigonometry and astronomy with several new tables, theorems, and observations, conceived the radius to be divided into 600,000 equal parts, and computed the sines of the arcs, for every 10 minutes, in such equal parts of the radius, by the decimal notation.

This project of Purbach was completed by his disciple, companion, and successor, John Muller, or Regiomontanus, being so called from the place of his nativity, the little town of Mons Regius, or Koningsberg, in Franconia, where he was born in the year 1436. Regiomontanus not only extended the sines to every minute, the radius being 600,000, as designed by Purbach, but afterwards disliking that scheme as evidently imperfect, he computed them also to the radius 1,000,000, for every minute of the quadrant. He also introduced the tangents into trigonometry, the canon of which he called *fœcundus*, because of the many and great advantages arising from them. Besides these, he enriched trigonometry with many theorems and precepts. Through the benefit of all these improvements, except for the use of logarithms, the trigonometry of Regiomontanus is but little inferior to that of our own time. His treatise, on both plane and spherical trigonometry, is in 5 books; it was written about the year 1464, and was printed in folio at Nuremberg, in 1533. And in the 5th book are also various problems concerning rectilinear triangles, some of which are resolved by means of algebra—a proof that this science was not wholly unknown in Europe before the treatise of Lucas de Burgo. Regiomontanus died in 1476, at the age of 40 years only; being then at Rome, whither he had been invited by the Pope, to assist in the reformation of the calendar, and where it was suspected he was poisoned by the sons of George Trebizonde, in revenge for the death of their father, which was said to have been caused by the grief he felt on account of the criticisms made by Regiomontanus on his translation of Ptolemy's Almagest.

Soon after this, several other mathematicians contributed to the improvement of trigonometry, by extending and enlarging the tables, though few of their works have been printed; and particularly John Werner of Nuremberg, who was born in 1468, and died in 1528, and who it seems wrote five books on triangles.

About the year 1500, Nicholas Copernicus, the celebrated modern restorer of the true solar system, wrote a brief treatise on trigonometry, both plane and spherical, with the description and construction of the canon of chords, or their halves, nearly in the manner of Ptolemy; to which is subjoined a canon of sines, with their differences, for every 10 minutes of the quadrant, to the radius 100,000. This tract is inserted in the first book of his "Revolutiones Orbium Cœlestium," first printed in folio at Nuremberg, 1543. It is remarkable that he does not call these lines *sines*, but *semisses subtensarum*, namely of the double arcs.—Copernicus was born at Thorn in 1473, and died in 1543.

In 1553 was published the "Canon Fœcundus," or table of tangents, of Erasmus Reinhold, professor of mathematics in the academy of Wurtemburg. He was born at Salfieldt in Upper Saxony, in the year 1511, and died in 1553.

To Francis Maurolyc, abbot of Messina in Sicily, we owe the introduction of the "Tabula Benefica," or canon of secants, which came out about the same time, or a little before. But Lansberg erroneously ascribes this to Rheticus. And the tangents and secants are both ascribed to Reinhold, by Briggs, in his "Mathematica ab antiquis minus cognita," (p. 30, Appendix to Ward's Lives of the Professors of Gresham College.)

Francis Vieta was born in 1540 at Fontenai, or Fontenaile-Comte, in Lower Poitou, a province of France. He was master of requests at Paris, where he died in 1603, being the 63d year of his age. Among other branches of learning in which he excelled, he was one of the most respectable mathematicians of the 16th century, or indeed of any age. His writings abound with marks of great originality, and the finest genius, as well as intense application. Among them are several pieces relating to trigonometry, which may be found in the collection of his works published at Leyden in 1646, by Francis Schooten, besides another large and separate volume in folio, published in the author's lifetime at Paris in 1579, containing trigonometrical tables, with their construction and use; very elegantly printed, by the king's mathematical printer, with beautiful types and rules; the differences of the sines, tangents and secants, and some other parts, being printed with red ink, for the better distinction; but it is inaccurately executed, as he himself testifies in page 323 of his other works above mentioned. The first part of this curious volume is entitled "Canon Mathematicus, seu ad Triangula, cum Appendicibus," and it contains a great variety of tables useful in trigonometry. The first of these is what he more peculiarly calls "Canon Mathematicus, seu ad Triangula," which contains all the sines, tangents, and secants for every minute of the quadrant, to the radius 100,000, with all their differences; and towards the end of the quadrant the tangents and secants are extended to 8 or 9 places of figures. They are arranged like our tables at present, increasing on the left-hand side to 45 degrees, and then returning upwards by the right hand side to 90 degrees; so that each number and its complement stand on the same line. But here the canon of what we now call tangents is denominated *fæcundus*, and that of the secants *fæcundissimus*. For the general idea prevailing in the form of these tables, is, not that the lines represented by the numbers are those which are drawn in and about a circle, as sines, tangents, and secants, but the three sides of right-angled triangles; this being the way in which those lines had always been considered, and which still continued for some time longer. Hence it is that he considers the canon as a series of plane right-angled triangles, one side being constantly 100,000; or rather as three series of such triangles, for he makes a distinct series for each of the three varieties, namely, according as the hypotenuse, or the base, or the perpendicular, is represented by the constant number 100,000, which is similar to the radius. Making each side constantly 100,000, the other two sides are computed to every magnitude of the acute angle at the base, from 1 minute up to 90 degrees, or the whole quadrant. Each of the three series therefore consists of two parts, representing the two variable sides of the triangle. When the hypotenuse is made the constant number 100,000, the two variable sides of the triangle are the perpendicular and base, or our sine and cosine; when the base is 100,000, the perpendicular and hypotenuse are the variable parts, forming the *canon fæcundus et fæcundissimus*, or our tangent and secant; and when the perpendicular is made the constant 100,000, the series contains the variable base and hypotenuse, or also *canon fæcundus et fæcundissimus*, or our cotangent and cosecant. Of course, therefore, the table consists of 6 columns, 2 for each of the three series, besides the two columns on the right and left for minutes, from 0 to 60 in each degree.

The second of these tables is similar to the first, but all in rational numbers, consisting, like *it*, of three series of two columns each; the radius, or constant side of the triangle, in each series, being 100,000, as before; and the other two sides *accurately* expressed in integers and rational vulgar fractions. So that we have here the canon of *accurate* sines, tangents, and secants, or a series of about 4300 rational right-angled triangles. But then the several corresponding arcs of the quadrant, or angles of those triangles, are not expressed. Instead of them, are inserted, in the first column next the margin, a series of numbers decreasing from the beginning to the end of the quadrant, which are called *numeri primi baseos*. It is from these numbers that Vieta constructs the sides of the three series of right-angled triangles, one side in each series being the constant number 100,000, as before. The theorems by which these series of rational triangles are computed from the *numeri primi baseos*, or marginal numbers, are inserted all in one page at the end of this second table, and in the modern notation they may be briefly expressed thus: Let p denote the primary or marginal number on any line, and r the constant radius or number 100,000; then if r denote the hypotenuse of the right-angled triangle, the perpendicular and base, or the sine and cosine will be respectively,

$\frac{pr}{\frac{1}{4}p^2+1}$ and $r - \frac{2r}{\frac{1}{4}p^2+1}$, (which last we may reduce to $\frac{\frac{1}{4}p^2-1}{\frac{1}{4}p^2+1}r$).
When r denotes the base of the right-angled triangle, the perpendicular and hypotenuse, or the tangent and secant, are expressed by

$\frac{pr}{\frac{1}{4}p^2-1}$ and $r + \frac{2r}{\frac{1}{4}p^2-1}$, (which last we may reduce to $\frac{\frac{1}{4}p^2+1}{\frac{1}{4}p^2-1}r$);
and when r denotes the perpendicular of the right-angled triangle, the base and hypotenuse, or the cotangent and cosecant, are then expressed by

$$\tfrac{1}{4}pr - \frac{r}{p} \text{ (or } \frac{\frac{1}{4}p^2-1}{p}r\text{), and } \tfrac{1}{4}pr + \frac{r}{p} \text{ (or } \frac{\frac{1}{4}p^2+1}{p}r\text{).}$$

So that Vieta's general values will be as we have here collected them together in the following expressions, immediately under the words sine, cosine, &c; and just below Vieta's forms I have here placed the others, to which they reduce and are equivalent, which are more contracted, though not so well adapted to the expeditious computation as Vieta's forms.

Sine	Cosine	Tangent	Secant	Cotangent	Cosecant
$\frac{pr}{\frac{1}{4}p^2+1}$	$r-\frac{2r}{\frac{1}{4}p^2+1}$	$\frac{pr}{\frac{1}{4}p^2-1}$	$r+\frac{2r}{\frac{1}{4}p^2-1}$	$\frac{1}{4}pr-\frac{r}{p}$	$\frac{1}{4}pr+\frac{r}{p}$
$\frac{p}{\frac{1}{4}p^2+1}r$	$\frac{\frac{1}{4}p^2-1}{\frac{1}{4}p^2+1}r$	$\frac{p}{\frac{1}{4}p^2-1}r$	$\frac{\frac{1}{4}p^2+1}{\frac{1}{4}p^2-1}r$	$\frac{\frac{1}{4}p^2-1}{p}r$	$\frac{\frac{1}{4}p^2+1}{p}r$

All these expressions, it is evident, are rational; and by assuming p of different values, from the first theorems Vieta computed the corresponding sides of the triangles, and so expressed them all in integers and rational fractions.

To the foregoing principal tables are subjoined several other smaller tables, or short specimens of large ones: as, a table of the sines, tangents and secants, for every single degree of the quadrant, with the corresponding lengths of the arcs, the radius being 100,000,000; another table of the sines, tangents, and secants, for each degree also, expressed in sexagesimal parts of the radius, as far as the third order of parts; also two other tables for the multiplication and reduction of sexagesimal quantities.

The second part of this volume is entitled "Universalium Inspectionum ad Canonem Mathematicum Liber singularis." It contains the construction of the tables, a compendious treatise on plane and spherical trigonometry, with the application of them to a great variety of curious subjects in geometry and mensuration, treated in a very learned manner; as also many curious observations concerning the quadrature of the circle, the duplication of the cube, &c. Computations are here given of the ratio of the diameter of a circle to the circumference, and of the length of the sine of 1 minute, both to many places of figures; by which he found that the sine of 1 minute is between 2,908,881,959 and 2,908,882,056; also, the diameter of a circle being 1000 &c, that the perimeter of the inscribed and circumscribed polygon of 393,216 sides, will be as follows:

perimeter of the inscrib. polygon 314,159,265,35,
perimeter of the circum. polygon 314,159,265,37,

and that therefore the circumference of the circle lies between those two numbers.

Though no author's name appears to the volume we have been describing, there can be no doubt of its being the performance of Vieta; for, besides bearing evident marks of his masterly hand, it is mentioned by himself in several parts of his other works collected by Schooten, and in the preface to those works by Elzevir, the printer of them; as also in Montucla's "Histoire des Mathematiques;" which are the only notices I have ever seen or heard of concerning this book, the copies of which are so rare, that I never saw one besides that which is in my own possession.

In the other works of Vieta, published at Leyden in 1646, by Schooten, as mentioned above, there are several other pieces of trigonometry; some of which, on account of their originality and importance, are very deserving of particular notice in this place. And first, the very excellent theorems, here first of all given by our author, relating to angular sections, the geometrical demonstrations of which are supplied by that ingenious geometrician, Alexander Anderson, then professor of mathematics at Paris, but a native of Aberdeen, and cousin-german to Mr. David Anderson, of Finzaugh, whose daughter was the mother of the celebrated Mr. James Gregory, inventor of the Gregorian telescope. We find here, theorems for the chords, and consequently sines, of the sums and differences of arcs; and for the chords of arcs that are in arithmetical progression, namely, that the 1st or least chord is to the 2d, as any one after the 1st is to the sum of the two next less and greater: for example, as the 2d to the sum of the 1st and 3d, and as the 3d to the sum of the 2d and 4th, and as the 4th to the sum of the 3d and 5th, &c; so that the 1st and 2d being given, all the rest are found from them by one subtraction, and one proportion for each, in which the 1st and 2d terms are constantly the same. Next are given theorems for the chords of any multiples of a given arc or angle, as also the chords of their supplements to a semicircle, which are similar to the sines and cosines of the multiples of given angles; and the conclusions from them are expressed in this manner: 1st, that if c be the chord of the supplement of a given arc a, to the radius 1; then the chords of the supplements of the multiple arcs will be as in the annexed table: where the author observes, that the signs are alternately + and −; that the vertical columns of numeral coefficients to the terms of the chords, are the several orders of figurate numbers, which he calls triangular, pyramidal, triangulo-triangular, triangulo-pyramidal, &c, *generated in the ordinary way by continual additions; not indeed from unity*, AS IN THE GENERATION OF POWERS, but beginning with the number 2; and that the powers observe always the same progression: secondly, that if the chord of an arc a be called 1, and d the chord of the double arc $2a$, then the chords of the series of multiple arcs will be as in this table; where the author remarks as before on the law of the powers, signs, and coefficients, these being the orders of figurate numbers, raised from unity by continual additions, *after the manner of the genesis of powers*, which generation in that way he speaks of as a thing generally known, but without giving any hint how the coefficients of the terms of any power may be found from one another only, and independent of those of any other power, as it was afterwards, and first of all, I believe, done by Henry Briggs, about the year 1600: and 3dly, that if c be the chord of any arc a, to the radius 1,

Arcs	Chords of the Sup.
$1a$	c
$2a$	c^2-2
$3a$	c^3-3c
$4a$	c^4-4c^2+2
$5a$	c^5-5c^3+5c
$6a$	$c^6-6c^4+9c^2-2$
$7a$	$c^7-7c^5+14c^3-7c$
&c.	&c.

Arcs	Chords.
$1a$	1
$2a$	d
$3a$	d^2-1
$4a$	d^3-2d
$5a$	d^4-3d^2+1
$6a$	d^5-4d^3+3d
$7a$	$d^6-5d^4+6d^2-1$
$8a$	$d^7-6d^5+10d^3-4d$
&c.	&c.

then the series of the chords and supplemental chords of the multiple arcs will be thus; where the values are alternately

Arcs	Chords and Chords of Sup.
1*a*	Chord $= + c$
2*a*	Sup. ch. $= - c^2 + 2$
3*a*	Chord $= - c^3 + 3c$
4*a*	Sup. ch. $= + c^4 - 4c^2 + 2$
5*a*	Chord $= + c^5 - 5c^3 + 5c$
6*a*	Sup. ch. $= - c^6 + 6c^4 - 9c^2 + 2$
7*a*	Chord $= - c^7 + 7c^5 - 14c^3 + 7c$
&c.	&c.

chords, and chords of the supplements of the arcs on the same line, and the law of the powers and coefficients as before, but every alternate couplet of lines having their signs changed.

Another curious theorem is added to the above, for finding the sum of all these chords drawn in a semicircle, from one end of the diameter to every point in the circumference, those points dividing the circumference into any number of equal parts; namely, as the least chord is to the diameter, so is the sum of the said least chord and diameter and greatest chord, to double the sum of all the chords, including the diameter as one of them.

As the above theorems are chiefly adapted for the chords of multiple angles, a few problems and remarks are then added (whether by Vieta or Anderson does not clearly appear, but I think by the latter) concerning the application of them, to the section of angles into submultiples, and thence to the computation of the chords or sines, or a canon of triangles. The general precept for the angular sections is this: select one of the above equations adapted to the proper number of the section, in which will be concerned the powers of the unknown or required quantity, as high as the index of the section; and from this equation find that quantity by the known methods for the resolution of equations. Examples are given of three different sections, namely, for 3, 5, and 7 equal parts, the forms of which are respectively these,

$$3c - c^3 \quad . \quad . \quad . \quad . \quad = g$$
$$5c - 5c^3 + c^5 \quad . \quad . \quad = g$$
$$7c - 14c^3 + c^5 - c^7 = g$$

where g is the chord of the given arc or angle, and c the required chord of the 3d, 5th, or 7th part of it. And it is shown, geometrically, that the first of these equations has 2 real positive roots, the second 3, and the last 4; also, from the same principles, the relations of these roots are pointed out.

The method then annexed for constructing the canon of sines, from the foregoing theorems is thus: By dividing the radius in extreme-and-mean ratio, is obtained the sine of 18 degrees; this quinquisected, gives the sine of 3° 36′. Again, by trisecting the arc of 60°, there is obtained the sine of 20°; this again trisected gives that of 6° 40′; and this bisected gives that of 3° 20′: Then, by the theorem for the difference of two arcs, there will be found the sine of 16′, the difference between 3° 36′ and 3° 20′: Lastly, by four successive bisections, will at length be found the sines of 8′, 4′, 2′, and 1′. This last being found, the sines of its multiples, and again of the multiples of these multiples, &c, throughout the quadrant, are to be taken by the proper theorems before laid down.—And the same subject is still further pursued and explained, in the tract containing the answer given by Vieta, to the problem proposed to the whole world by Adrianus Romanus. In the same collection of Vieta's works, from page 400 to 432, is given a complete treatise on practical trigonometry, containing rules for resolving all the cases of plane and spherical triangles, by the *Canon Mathematicus*, or table of sines, tangents and secants.

The next authors whose labours in this way have been printed, are Rheticus, Otho, and Pitiscus: to all of whom we owe very great improvements in trigonometry.—George Joachim Rheticus, professor of mathematics in the university of Wittemberg, and sometime pupil to Copernicus, died in 1576, in the 60th year of his age. He conceived, and executed, the great design of computing the triangular canon for every 10 seconds of the quadrant, to the radius 1000000000000000, consisting of 1, followed by 15 ciphers. The series of sines which Rheticus computed to this radius, for every 10 seconds, and for every single second in the first and last degree of the quadrant, was published in folio at Francfort, 1613, by Pitiscus, who himself added a few of the first sines computed to the radius 1000000000000000000000.

But the large work, or whole trigonometrical canon computed by Rheticus, was published in 1596 by Valentine Otho, mathematician to the Electoral Prince Palatine. This vast work contains all the three series for the whole canon of right-angled triangles (being similar to the sines, tangents and secants, by which names I shall call them), with all the differences of the numbers, to the radius 10000000000.

Prefixed to these tables, are several books on their construction and use, in plane and spherical trigonometry, &c. Of these, the first three are by Rheticus himself; namely, book the 1st, containing the demonstrations of 9 lemmas, concerning the properties of certain lines drawn in and about circles: the 2d book contains 10 propositions, relating to the sines and cosines of arcs, together with those of their sums and differences, their halves and doubles, &c. The 3d book teaches, in 13 propositions, the construction of the canon to the radius 1000000000000000. By some of the common properties of geometry, having determined the sines of a few principal arcs, as 30°, 36°, &c, in the first proposition, by continual bisections, he finds the sines of various other arcs, down to 45 minutes. Then, in the 2d proposition, by the theorems for the sums and differences of arcs, he finds all the sines and cosines, up to 90 degrees, in a series of arcs differing by 1° 30′. And, in the 3d proposition, by the continual addition of 45′, he obtains all the sines and cosines in the series whose common difference is 45′. In the 4th proposition, beginning with 45′, and continually bisecting, he finds the sines and cosines of the series of half arcs, till he arrives at the arc of 14^{viii} 19^{ix}, the sine of which is found to be 1, and its cosine 999999999999999. In the 5th proposition are computed the sine and cosine of 30″, or half a minute. In the 6th and 7th propositions are computed the sines and cosines for every minute, from 1′ to 45′, as well as of many larger arcs. The 8th proposition extends the computation for single minutes much farther. In propositions 9 and 10 are computed the tangents and secants for all arcs in the series whose common difference is 45′; and these are deduced from the sines of the same arcs by one proportion for each. In the remaining three propositions, 11, 12, 13, are computed the tangents and secants for several small angles. And from all these primary sines, tangents, and secants, the whole canon is deduced and completed.

The remaining books in this work are by the editor Otho; namely, a treatise, in one book, on right-angled plane triangles, the cases of which are resolved by the tables: then right-angled spherical trigonometry, in four books; next oblique spherical trigonometry, in five books; and lastly several other books, containing various spherical problems.

Next after the above are placed the tables themselves, containing the sines, tangents, and secants, for every 10 seconds in the quadrant, with all the differences annexed to each, in a smaller character. The numbers however are not called sines, tangents, and secants, but, like Vieta's, before described, they are considered as representing the sides of right-angled triangles, and are titled accordingly. They are also, in like manner, divided into three series, namely, according as the radius, or constant side of the triangle, is made the hypotenuse, or the greater leg, or the less leg of the triangle. When the hypotenuse is made the constant radius 10000000000, the two columns of this case, or series, are called the perpendicular and base, which are our sine and cosine; when the greater leg is the constant radius, the two columns on this series are titled hypotenuse and perpendicular, which are our secant and tangent; and when the less leg is constant, the two columns in this case are called hypotenuse and base; which are our cosecant and cotangent. After this large canon, is printed another smaller table, which is said to be the two columns of the third series, or cosecants and cotangents, with their differences, but to 3 places of figures less, or to the radius 10000000. But I cannot discover the reason for adding this less table, even if it were correct, which is very far from being the case, the numbers being uniformly erroneous, and different from the former through the greatest part of the table.

Towards the close of the 16th century, many persons wrote on the subject of trigonometry, and the construction of the triangular canon. But, their writings being seldom printed till many years afterwards, it is not easy to assign their order in respect of time. I shall therefore mention but a few of the principal authors, and that without pretending to any great precision on the score of chronological precedence.

In 1591 Philip Lansberg first published his "Geometria Triangulorum," in four books, with the canon of sines, tangents, and secants; a brief, but very elegant work; the whole being clearly explained: and it is perhaps the first set of tables titled with those words. The sines, tangents, and secants of the arcs to 45 degrees, with those of their complements, are each placed in adjacent columns, in a very commodious manner, continued forwards and downwards to 45 degrees, and then returning backwards and upwards to 90 degrees: the radius is 10000000, and a specimen of the first page of the table is as follows:

0	Sinus		Tangens		Secans		
0	0	10000000	0	infinitum.	10000000	infinitum.	60
1	2909	9999999	2909	34377466738	10000000	34377468193	59
2	5818	9999998	5818	17188731915	10000002	17188734824	58
3	8727	9999996	8727	11459152994	10000004	11459157357	57
4	11636	9999993	11636	8594363048	10000007	8594368866	56
5	14544	9999989	14544	6875488693	10000011	6875495966	55
&c.							&c.
							89

Of this work, the first book treats of the magnitude and relations of such lines as are considered in and about the circle, as the chords, sines, tangents, and secants. In the second book is delivered the construction of the trigonometrical canon, by means of the properties laid down in the first book: After which follows the canon itself. And in the third and fourth books is shown the application of the table, in the resolution of plane and spherical triangles.—Lansberg, who was born in Zealand 1561, was many years a minister of the gospel, and died at Middleburg in 1632.

The trigonometry of Bartholomew Pitiscus was first published at Francfort in the year 1599. This is a very complete work; containing, besides the triangular canon, with its construction and use in resolving triangles, the application of trigonometry to problems of surveying, altimetry, architecture, geography, dialling, and astronomy. The construction of the canon is very clearly described: And, in the third edition of the book, in the year 1612, he boasts to have added, in this part, arithmetical rules for finding the chords of the 3d, 5th, and other uneven parts of an arc, from the chord of that arc being given; saying, that it had been heretofore thought impossible to give such rules: But, after all, those boasted methods are only the application of the double rule of False-Position to the then known rules for finding the chords of multiple arcs; namely, making the supposition of some number for the required chord of a submultiple of any given arc, then from this assumed number computing what will be the chord of its multiple arc, which is to be compared with that of the given arc; then the same operation is performed with another supposition; and so on, as in the double rule of position. The canon contains the sine, tangent, and secant, for every minute of the quadrant, in some parts to 7 places of figures, in others to 8; as also the differences for every 10 seconds. The sines, tangents, and secants, are also given for every 10 seconds in the first and last degree of the quadrant, for every 2 seconds in the first and last 10 minutes, and for every single second in the first and last minute. In this table, the sines, tangents, and secants, are continued downwards on the left-hand pages, as far as to 45 degrees, and then returned upwards on the right-hand pages, so that the complements are always on the same line in the opposite or facing pages.

The mathematical works of Christopher Clavius (a German jesuit, who was born at Bamberg in 1537) in five large folio volumes, were printed at Moguntia, or Mentz, in 1612, the year in which the author died, at the age of 75. In the first volume we find a very ample and circumstantial treatise on trigonometry, with Regiomontanus's canon of sines, for every minute, as also canons of tangents and secants, each in a separate table, to the radius 10000000, and in a form continued forwards all the way up to 90 degrees. The explanation of the construction of the tables is very complete, and is chiefly extracted from Ptolemy, Purbach, and Regiomontanus. The sines have the differences set down for each second, that is, the quotients arising from the differences of the sines divided by 60.

About the year 1600, Ludolph van Collen, or à Ceulen, a respectable Dutch mathematician, wrote his book " de circulo et adscriptis," in which he treats fully and ably of the properties of lines drawn in and about the circle, and especially of chords or subtenses, with the construction of the canon of sines. The geometrical properties from which these lines are computed, are the same as those used by former writers; but his mode of computing and expressing them is different from theirs; for they actually extracted all the roots, &c, at every step, or single operation, in decimal numbers; but he retained the radical expressions to the last, making them however always as simple as possible: thus, for instance, he determines the sides of the polygons of 4, 8, 16, 32, &c, sides, inscribed in the circle whose radius is 1, to be as in the table here annexed: where the point before any figure, as $\surd .2$ signifies the

No. of Sides.	Length of each side.
4	$\surd 2$
8	$\surd .2 - \surd 2$
16	$\surd .2 - \surd .2 + \surd 2$
32	$\surd .2 - \surd .2 + \surd .2 - \surd 2$
&c.	&c.

root of all that follows it; so the last line is in our notation the same as $\sqrt{2-\sqrt{2+\sqrt{2-\sqrt{2}}}}$. And as the perfect management of such surds was then not generally known, he added a very neat tract on that subject, to facilitate the computations. These, together with other dissertations on similar geometrical matters, were translated from the Dutch language, into Latin, by Willebrord Snell, and published at (Lugd. Batav.) Leyden in 1619. It was in this work that Ludolph determined the ratio of the diameter to the circumference of the circle, to 36 figures, showing that, if the diameter be 1, the circumference will be

greater than 3·14159 26535 89793 23846 26433 83279 50288,
but less than 3·14159 26535 89793 23846 26433 83279 50289,

which ratio was, by his order, in imitation of Archimedes, engraven on his tomb-stone, as is witnessed by the said Snell, pa. 54, 55, "Cyclometricus," published at Leyden two years after, in which he treats the same subject in a similar manner, recomputing and verifying Ludolph's numbers. And, in the same book, he also gives a variety of geometrical approximations, or mechanical solutions, to determine very nearly the lengths of arcs, and the areas of sectors and segments of circles.

Besides the "Cyclometricus," and another geometrical work (Apollonius Battavus) published in 1608, the same Snell wrote also four others "doctrinæ triangulorum canonicæ," in which is contained the canon of secants, and in which the construction of sines, tangents, and secants, together with the dimension or calculation of triangles, both plane and spherical, are briefly and clearly treated. After the author's death, this work was published in 8vo, at Leyden, 1627, by Martinus Hortensius, who added to it a tract on surveying and spherical problems. Willebrord Snell was born in 1591 at Royen, and died in 1626, being only 35 years of age. He was professor of mathematics in the university of Leyden, as was also his father Rodolph Snell.

Also in 1627, Francis van Schooten published, at Amsterdam, in a small neat form, tables of sines, tangents, and secants, for every minute of the quadrant, to 7 places of figures, the radius being 10000000; together with their use in plane trigonometry. These tables have a great character for their accuracy, being declared by the author to be without one single error. This however must not be understood of the last figure of the numbers, which I find are very often erroneous, sometimes in excess and sometimes in defect, by not being always set down to the nearest unit. Schooten died in 1659, while the second volume of his second edition of Descartes' geometry was in the press. He was also author of several other valuable works in geometry, and other branches of the mathematics.

The foregoing are the principal writers on the tables of sines, tangents, and secants, before the invention of logarithms, which happened about this time, namely, soon after the year 1600. Tables of the natural numbers were now all completed, and the methods of computing them nearly perfected: And therefore, before entering on the discovery and construction of logarithms, I shall stop here awhile to give a summary of the manner in which the said natural sines, tangents, and secants, were actually computed, after having been gradually improved from Hipparchus, Menelaus, and Ptolemy, who used only the chords, down to the beginning of the 17th century, when sines, tangents, secants, and versed sines were in use, and when the method hitherto employed had received its utmost improvement. In this explanation, we may here first enumerate the theorems by which the calculations were made, and then describe the application of them to the computation itself.

Theorem 1.—The square of the diameter of a circle, is equal to the sum of the squares of the chord of an arc, and of the chord of its supplement to a semicircle.—2. The rectangle under the two diagonals of any quadrilateral inscribed in a circle, is equal to the sum of the two rectangles under the opposite sides.—3. The sum of the squares of the sine and cosine, hitherto called the sine of the complement, is equal to the square of the radius.—4. The difference between the sines of two arcs that are equally distant from 60 degrees, or $\frac{1}{6}$ of the whole circumference, the one as much greater as the other is less, is equal to the sine of half the difference of those arcs, or of the difference between either arc and the said arc of 60 degrees.—5. The sum of the cosine and versed sine, is equal to the radius.—6. The sum of the squares of the sine and versed sine, is equal to the square of the chord, or to the square of double the sine of half the arc.—7. The sine is a mean proportional between half the radius and the versed sine of double the arc.—8. A mean proportional between the versed sine and half the radius, is equal to the sine of half the arc.—9. As radius is to the sine, so is twice the cosine to the sine of twice the arc.—10. As the chord of an arc, is to the sum of the chords of the single and double arc, so is the difference of those chords, to the chord of thrice the arc.—11. As the chord of an arc, is to the sum of the chords of twice and thrice the arc, so is the difference of those chords, to the chord of five times the arc.—12. And in general, as the chord of an arc, is to the sum of the chords of n times and $n + 1$ times the arc, so is the difference of those chords, to the chord of $2n + 1$ times the arc.—13. The sine of the sum of two arcs, is equal to the sum of the products of the sine of each multiplied by the cosine of the other, and divided by the radius.—14. The sine of the difference of two arcs, is equal to the difference of the said two products divided by radius.—15. The cosine of the sum of two arcs, is equal to the difference between the products of their sines and of their cosines, divided by radius.—16. The cosine of the difference of two arcs, is equal to the sum of the said products divided by radius.—17. A small arc is equal to its chord or sine, nearly.—18. As cosine is to sine, so is radius to tangent.—19. Radius is a mean proportional between the tangent and cotangent.—20. Radius is a mean proportional between the secant and cosine.—21. Radius is a mean proportional between the sine and cosecant.—22. Half the difference between the tangent and cotangent of an arc, is equal to the tangent of the difference between the arc and its complement. Or, the sum arising from the addition of double the tangent of an arc with the tangent of half its complement, is equal to the tangent of the sum of that arc and the said half complement.—23. The square of the secant of an arc, is equal to the sum of the squares of the radius and tangent.—24. The secant of an arc, is equal to the sum of its tangent and the tangent of half its complement. Or, the secant of the difference between an arc and its complement, is equal to the tangent of the said difference added to the tangent of the less arc.—25. The secant of an arc, is equal to the difference between the tangent of that arc and the tangent of the arc added to half its complement. Or, the secant of the difference between an arc and its complement, is equal to the difference between the tangent of the said difference and the tangent of the greater arc.

From some of these 25 theorems, extracted from the writers before mentioned, and a few propositions of Euclid's elements, they compiled the whole table of sines, tangents, and secants, nearly in the following manner. By the elements were computed the sides of a few of the regular figures inscribed in a circle, which were the chords of such parts of the whole circumference as are expressed by the number of sides, and therefore the halves of those chords the sines of the halves of the arcs. So, if the radius be 10000000, the sides of the following figures will give the annexed chords and sines.

The figure.	Arcs subtended	Its chord or side.	Half arc.	Its sine or ½ chord.
Triangle	120°	17320508	60°	8660254
Square	90	14142136	45	7071068
Pentagon	72	11755705	36	5877853
Hexagon	60	10000000	30	5000000
Decagon	36	6180340	18	3090170
Quindecagon	24	4158234	12	2079117

Of some, or all of these, the sines of the halves were continually taken, by theorem the 6th, 7th, or 8th, and of their complements by the 3d; then the sines of the halves of these, and of their complements, by the same theorems; and so on, alternately, of the halves and complements, till they arrived at an arc which is nearly equal to its sine. Thus, beginning with the above arc of 12 degrees, and its sine, the halves were obtained as follows:

The halves.		Sines.
6°	′	1045285
3		523360
1	30	261769
	45	130896
The Comp. of these.		
84		9945218
87		9986295
88	30	9996573
89	15	9999143
The halves of these.		
42		6691306
21		3583679
10	30	1822355
5	15	915016
43	30	6883545
21	45	3705574
44	15	6977905

The comp. of these.		Sines.
48°	′	7431448
69		9335804
79	30	9832549
84	45	9958049
46	30	7253744
68	15	9288095
45	45	7163019
The halves of these.		
24		4067366
34	30	5664062
17	15	2965416
39	45	6394390
23	15	3947439
The comp.		
66		9135455
55	30	8241262
72	45	9550199
50	15	7688418
66	45	9187912

The halves.		Sines.
33°	′	5446390
16	30	2840153
8	15	1434926
27	45	4656145
Comps.		
57		8386706
73	30	9588197
81	45	9896514
62	15	8849876
Halves.		
28	30	4771588
14	15	2461533
36	45	5983246
Comps.		
61	30	8788171
75	45	9692309
53	15	8012538
Half.		
30	45	5112931
Comp.		
59	15	8594064

The sines of small arcs are then deduced in this manner. From the sine of 45′, above determined, are found the halves, which will be thus:

45′	0″	- - - - -	130896
22	30	- - - - -	65449,4
11	15	- - - - -	32724,8

Now these last two sines being evidently in the same ratio as their arcs, the sines of all the less single minutes will be found by single proportion. So the 45th part of the sine of 45′, gives 2909 for the sine of 1′; which may be doubled, tripled, &c, for the sines of 2′, 3′, &c, up to 45′.

Then, from all the foregoing primary sines, by the theorems for halving, doubling, or tripling, and by those for the sums and differences, the rest of the sines are deduced, to complete the quadrant.

But having thus determined the sines and cosines of the first 30° of the quadrant, that is, the sines of the first and last 30°, those of the intermediate 30° are, by theor. 4, found by one single subtraction for each sine.

The sines of the whole quadrant being thus completed, the tangents are found by theor. 18, 19, 22, namely, for one half of the quadrant by the 18th and 19th, and the other half, by one single addition or subtraction for each, by the 22d theorem. And lastly, by theor. 24 and 25, the secants are deduced from the tangents, by addition and subtraction only.

Among the various means used for constructing the canon of sines, tangents, and secants, the writers above enumerated seem not to have been possessed of the method of differences, so profitably used since, and first of all I believe by Briggs, in computing his trigonometrical canon and his logarithms, as we shall see hereafter, when we come to describe those works. They took however the successive differences of the numbers, after they were computed, to verify or prove the truth of them; and if found erroneous, by any irregularity in the last differences, from thence they had a method of correcting the original numbers themselves. At least, this method is used by Pitiscus, Trig. lib. 2, where the differences are extended to the third order.—In page 44 of the same book also is described, for the first time that I know of, the common notation of decimal fractions, as now used. And this same notation was afterwards described and used by baron Napier, in positio 4 and 5 of his posthumous works, on the construction of logarithms, published by his son in the year 1619. But the decimal fractions themselves may be considered as having been introduced by Regiomontanus, by his decimal division of the radius, &c, of the circle; and from that time gradually brought into use; but continued long to be denoted after the manner of vulgar fractions, by a line drawn between the numerator and denominator, which last however was soon omitted, and only the numerator set down, with the line below it: thus, it was first $31\frac{35}{100}$, then $31\underline{35}$; afterwards, omitting the line, it became 31^{35}, and lastly $31_{,35}$, or 31.35, or 31·35: as may be traced in the works of Vieta, and others since his time, gradually into the present century.

Having often heard it remarked, that the word *sine*, or in Latin and French *sinus*, is of doubtful origin; and as the various accounts which I have seen of its derivation are very different from one another, it may not be amiss here to employ a few lines on this matter. Some authors say, this is an Arabic word, others that it is the single Latin word *sinus*; and in Montucla's "Histoire des Mathematiques" it is conjectured to be an abbreviation of two Latin words. The conjecture is thus expressed by the ingenious and learned author of that excellent history, at p. xxxiii, among the additions and corrections of the first volume: "A l'occasion des sinus dont on parle dans cette page, comme d'une invention des Arabes, voici une étymologie de ce nom, tout-a-fait heureuse et vraisemblable. Je la dois à M. Godin, de l'Académie Royale des Sciences, Directeur de l'Ecole de Marine de Cadix. Les sinus sont, comme l'on scait, des moitiés de cords; et les cordes en Latin se nomment *inscriptæ*. Les sinus sont donc *semisses inscriptarum*, ce que probablement on écrivit ainsi pour abréger, S. Ins. Delà ensuite s'est fait par abus le mot de sinus." Now, ingenious as this conjecture is, there appears to be little or no probability for the truth of it. For, in the first place, it is not in the least supported by quotations from any of the more early books, to show that it ever was the practice to write or print the words thus, *S. Ins.* upon which the conjecture is founded. Again, it is said the chords are called in Latin *inscriptæ*; and it is true that they sometimes are so: but I think they are more frequently called *subtensæ*, and the sines *semisses subtensarum* of the double arcs, which will not abbreviate into the word *sinus*. This conjecture the learned author has relinquished in the new edition of his history. But it may be said, what reason have we to suppose that this word is either a Latin word, or the abbreviation of any Latin words whatever? and that it seems but proper to seek for the etymology of *words* in the language of the inventors of the *things*. For which reason it is, that we find the two other words, *tangens* and *secans*, are Latin, as they were invented and used by authors who wrote in that language. But the sines are acknowledged to have been invented and introduced by the Arabians, and thence by analogy it would seem probable that this is a word of *their* language, and from them adopted, together with the use of it, by the Europeans. And indeed Lansberg, in the second page of his trigonometry abovementioned, expressly says, that it *is* Arabic: His words are, *Vox sinus Arabica est, et proinde barbara; sed cum longo usu approbata sit, et commodior non suppetat, nequaquam repudianda est: faciles enim in verbis nos esse oportet, cùm de rebus convenit.* And Vieta says something to the same purport, in page 9 of his " Universalium Inspectionum ad Canonem Mathematicum Liber:" His words are, *Breve sinus vocabulum, cùm sit artis, Saracenis præsertim quàm familiare, non est ab artificibus explodendum, ad laterum semissium inscriptorum denotationem, &c.*

Guarinus also is of the same opinion: in his " Euclides Adauctus," &c. tract xx. pa. 307, he says, SINUS *vero est nomen Arabicum usurpatum in hanc significationem à mathematicis;* though he was aware that a Latin origin was ascribed to it by Vitalis, for he immediately adds, *Licet Vitalis in suo Lexico Mathematico ex eo velit sinum appellatum, quòd claudat curvitatem arcus.*

Long before I either saw or heard of any conjecture, or observation, concerning the etymology of the word *sinus*, I remember that I *imagined* it to be taken from the same Latin word, signifying breast or bosom, and that our sine was so called allegorically. I had observed, that several of the terms in trigonometry were derived from a bow to shoot with, and its appendages; as *arcus* the bow, *chorda* the string, and *sagitta* the arrow, by which name the versed sine, which represents it, was sometimes called; also, that the *tangens* was so called from its office, being a line *touching* the circle, and *secans* from its *cutting* the same: I therefore imagined that the *sinus* was so called, either from its resemblance to the breast or bosom, or from its being a line drawn within the bosom *(sinus)* of the arc, or from its being that part of the string *(chorda)* of a bow *(arcus)* which is drawn near the breast *(sinus)* in the act of shooting. And perhaps Vitalis's definition, above-quoted, has some allusion to the same similitude.

Also Vieta seems to allude to the same thing, in calling *sinus* an allegorical word, in page 417 of his works, as published by Schooten, where, with his usual judgment and precision, he treats of the propriety of the terms used in trigonometry for certain lines drawn in and about the circle; of which, as it very well deserves, I shall here extract the principal part, to show the opinion and arguments of so great a man on those names. "Arabes autem semisses inscriptas duplo, numeris præsertim æstimatas, vocaverunt allegoricè SINUS, atque ideo ipsam semi-diametrum, quæ maxima est semissium inscriptarum, SINUM TOTUM. Et de iis suâ methodo canones exiverunt qui circumferuntur, supputante præsertim Regiomontano benè justè et accuratè, in iis etiam particulis qualium semidiameter adsumitur 10,000,000.

"Ex canonibus deinde sinuum derivaverunt recentiores canonem semissium circumscriptarum, quem dixêre *Fæcundum;* et canonem eductarum è centro, quem dixêre *Fæcundissimum* et *Beneficum*, hypotenusis addictum. Atque adeò semisses circumscriptas, numeris præsertim æstimatas, vocaverunt *Fæcundos*, Sinus numeròsve videlicet; quanquam nihil vetat *Fæcundi* nomen substantivè accipi. Hypotenusas autem Beneficas, vel etiam simpliciter Hypotenusas: quoniam hypotenusa in primâ serie sinûs totius nomen retinet. Itaque ne novitate verborum res adumbretur, et alioqui sua artificibus, eo nomine dibita, præripiatur gloria, præposita in Canone Mathematico canonicis numeris inscriptio, candidè admonet primam seriem esse Canonem Sinuum. In secundâ vero, partem canonis fœcundi, partem canonis fœcundissimi, cotineri. In tertiâ, reliquam.

" Sanè præter inscriptas et circumscriptas, circulum etiàm adficiunt aliæ lineæ rectæ, velut Incidentes, Tangentes, et Secantes. Verùm illæ voces substantivæ sunt, non peripheriarum relativæ. Ac secare quidem circulum linea recta tunc intelligitur, cum in duobus punctis secat. Itaque non loquuntur benè geometricè, qui eductas è centro ad metas circumscriptarum vocant secantes impropriě, cum secantes et tangentes ad certos angulos vel peripherias referunt. Immò verò artem confundunt, cum his vocibus necesse habeat uti geometra abs relatione.

" Quare si quibus arrideat Arabum metaphora; quæ quidem aut omninò retinenda videtur, aut omninò explodenda; ut semisses inscriptas, Arabes vocant sinus; sic semisses circumscriptæ, vocentur Prosinus Amsinusve; et eductæ è centro Transinuosæ. Sin allegoria displiceat, geometrica sane inscriptarum et circumscriptarum nomina retineantur. Et cum eductæ è centro ad metas circumscriptarum, non habeant hactenus nomen certum neque elegans, voceantur sanè prosemidiametri, quasi protensæ semidiametri, se habentes ad suas circumscriptas, sicut semidiametri ad inscriptas."

Against the Arabic origin however of this word *(sinus)* may be urged its being varied according to the fourth declension of Latin nouns, like *manus;* and that if it were an Arabic word latinized, it would have been ranked under either the first, second, or third declension, as is usual in such adopted words.

So that, upon the whole, it will perhaps rather seem probable, that the term *sinus* is the Latin word answering to the name by which the Saracens called that line, and not their word itself. And this conjecture seems to be rendered still more probable by some expressions in pa. 4 and 5 of Otho's " Preface to Rheticus's Canon," where it is not only said, that the Saracens called the half-chord of double the arc *sinus*, but also that they called the part of the radius lying between the sine and the arc *sinus versus, vel sagitta*, which are evidently Latin words, and seem to be intended for the Latin translations of the names by which the Arabians called these lines, or the numbers expressing the lengths of them.

And this conjecture has been confirmed and realised, by a reference to Golius's Lexicon of the Arabic and Latin languages. In consequence I find that the Arabic and Latin writers on trigonometry do both of them use those words in the same allegorical sense, the latter being the Latin translations of the former, and not the Arabic words corrupted. Thus, the true Arabic word to denote the trigonometrical sine is جيب, pronounced *Jeib*, (reading the vowels in the French manner), meaning *sinus indusii, vestisque*, the bosom part of the garment; the versed sine is سهم, *Sehim*, which is *sagitta*, the arrow; the arc is قوس, which is *arcus*, the arc; and the chord is وتر, *Vitr*, that is *chorda*, the chord.

TRACT XX.

HISTORY OF LOGARITHMS.

THE trigonometrical canon, of natural sines, tangents, and secants, being now brought to a considerable degree of perfection; the great length and accuracy of the numbers, together with the increasing delicacy and number of astronomical problems, and spherical triangles, to the solution of which the canon was applied, urged many persons, conversant in those matters, to endeavour to discover some means of diminishing the great labour and time, requisite for so many multiplications and divisions, in such large numbers as the tables then consisted of. And their chief aim was, to reduce the multiplications and divisions to additions and subtractions, as much as possible.

For this purpose, Nicholas Raymer Ursus Dithmarsus invented an ingenious method, which serves for one case in the