5. $r = \frac{2abc}{\sqrt{(a^2b^2 - (a^2 + b^2 - c^2)^2)}}$

$= \frac{2abc}{\sqrt{[(a+b+c)\times(a+b-c)\times(a-b+c)\times(-a+b+c)]}}$.

6. $c = \sqrt{[a^2 + b^2 - 2ab\sqrt{(1 - (\frac{\text{C}}{n} - \frac{\text{C}^3}{2.3n^3} - \frac{\text{C}^5}{2.3.4.5n^5}\&\text{c})^2}]}$.

Where a, b, c, are the halves of the three sides of the triangle, and A the number of degrees in the angle opposite the side $2a$, and C the degrees in the angle opposite the side $2c$; also r is the radius of the circumscribed circle;

and $n = \frac{180}{3{\cdot}14159} = 57{\cdot}2957795$, or $\frac{n}{105} = {\cdot}54567409$.

EXAMPLE.

Thus, if the three sides be given, as for example $a = 13$, $b = 14$, $c = 15$. Then is $r = 16\frac{1}{4}$, and the angles by these theorems come out as follow; viz.

Angles by the Theor.			The true Angles.	
53°	7′	- - angle A - -	53°	$7'\frac{4}{5}$
59	28	- - angle B - -	59	$29\frac{2}{5}$
67	19	- - angle C - -	67	$22\frac{4}{5}$
179	54	sum of all	180	00

TRACT XVII,

ON MACHIN'S QUADRATURE OF THE CIRCLE.

SINCE the chief advantage of this method consists in taking small arcs, whose tangents shall be numbers easy to manage, Mr. Machin very properly considered, that as the tangent of 45° is 1; and that the tangent of any arc being given, the tangent of double that arc can easily be found; if there be assumed some small simple number for the tangent of an arc, and then the tangent of the double arc be continually taken, till a tangent be found nearly equal to 1, the tangent of 45°, by taking the tangent answering to the small difference between 45° and this multiple, there would be obtained two very small tangents, viz. the tangent first assumed, and the tangent of the difference between 45° and the multiple arc; and that therefore the lengths of the arcs corresponding to these two tangents being calculated, and the arc belonging to the tangent first assumed being as often doubled as the multiple denotes, the result increased or diminished by the other arc, would be the arc of 45°, according as the multiple arc should be below or above it.

Having thus thought of his method, by a few trials he was lucky enough to find a number, and perhaps the only one, proper for this purpose, viz, knowing that the tangent of $\frac{1}{4}$ of 45° is nearly $= \frac{1}{5}$, he assumed $\frac{1}{5}$ as the tangent of an arc: then since, if t be the tangent of an arc, the tangent of the double arc will be $\frac{2t}{1-tt}$ the radius being 1; the tangent of an arc double to that of which $\frac{1}{5}$ is the tangent, will be $\frac{\frac{2}{5}}{1-\frac{1}{25}} = \frac{10}{24} = \frac{5}{12}$, and the tangent of the double of this last is $\frac{\frac{10}{12}}{1-\frac{25}{144}} = \frac{120}{199}$; which, being very near equal to 1, shows that the arc which is equal to 4 times the first, is very near 45°. Then, since the tangent of the difference between 45° and an arc whose tangent is T, is $\frac{T-1}{T+1}$, we shall have the tangent of the difference between 45° and the arc whose tangent is $\frac{120}{119}$ equal to $\frac{\frac{120}{119}-1}{\frac{120}{119}+1} = \frac{120-119}{120+119} = \frac{1}{239}$.

Now by calculating, from the general series, the arcs whose tangents are $\frac{1}{5}$ and $\frac{1}{239}$, which may be quickly done, by reason of the smallness and the simplicity of the numbers, and taking the latter arc from 4 times the former, the remainder will be the arc of 45°. And this is Mr. Machin's ingenious quadrature of the circle.

But it was by means of Dr. Halley's method that Mr.

Machin found the circumference of a circle, whose diameter is 1, to be

3·14159265335,8979323846,2643383279,5028841971,6939937510,
5820974944,5923078164,0628620899,8628034825,3421170679 +,

true to above 100 places of figures.

Or, by substituting the above numbers in Machin's series, we get the series $(\frac{16}{5}-\frac{4}{239})-\frac{1}{3}(\frac{16}{5^3}-\frac{4}{239^3})+\frac{1}{5}(\frac{16}{5^5}-\frac{4}{239^5}$ &c, equal to the semicircumference whose radius is 1, or the whole circumference whose diameter is 1. Being the series published by Mr. Jones, and which he acknowledges he received from Mr. Machin.

But because the arc whose tangent is $\frac{1}{5}$, is $=$ 2 times the arc whose tangent is $\frac{1}{10}$, minus the arc to tangent $\frac{1}{515}$; (for $\frac{\frac{2}{10}}{1-\frac{1}{100}}=\frac{20}{99}=$ tangent of twice the arc to tangent $\frac{1}{10}$, and $\frac{\frac{20}{99}-\frac{1}{5}}{1+\frac{4}{99}}=\frac{1}{515}=$ tang. of diff. between the arcs whose tangents are $\frac{20}{99}$ and $\frac{1}{5}$); therefore 8 times arc to tangent $\frac{1}{10}-4$ times arc to tang. $\frac{1}{515}$ − arc to tang. $\frac{1}{239}=$ arc of 45°, or whose tang. is 1. Which is much easier than Machin's way. And various other methods may easily be discovered from the same principles.

TRACT XVIII.

A NEW AND GENERAL METHOD OF FINDING SIMPLE AND QUICKLY-CONVERGING SERIES; BY WHICH THE PROPORTION OF THE DIAMETER OF A CIRCLE TO ITS CIRCUMFERENCE MAY EASILY BE COMPUTED TO A GREAT MANY PLACES OF FIGURES.

In examining the methods of Mr. Machin and others, for computing the proportion of the diameter of a circle to its circumference, I discovered the method explained in this paper. This method is very general, and discovers many