would still hold good, if AB were any other diameter of the ellipse, instead of the axis; describing on the parts of it semi-ellipses which shall be similar to those into which the diameter AB divides the given ellipse.

10. And further, if a circle be described about the ellipse, on the diameter AB, and lines be drawn similar to those in the second figure; then, by a process the very same as in Art. 4, *et seq.* substituting only semiellipse for semicircle, it is found that the space

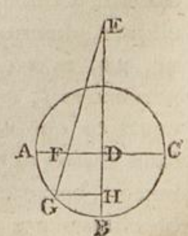
PQ is equal to the similar ellipse on the diameter BE,
PQRS is equal to the similar ellipse on the diameter BF,
RS is equal to the similar ellipse on the diameter AH,
or to the difference of the ellipses on BF and BE;
also the elliptic spaces - - - PQ, PQRS, RS, TV,
are respectively as the lines - BC, BD, DC, AD,

the same ratio as the circular spaces. And hence an ellipse is divided into any number of parts, in any assigned ratios, in the same manner as the circle is divided, namely, dividing the axis, or any diameter in the same manner, and on the parts of it describing similar semiellipses.

TRACT XV.

AN APPROXIMATE GEOMETRICAL DIVISION OF THE CIRCLE.

THE solution, here improved, of the following problem, I first gave in my Miscellanea Mathematica, published in the year 1775, pa. 311. The problem is as follows.



To find whether there is any such fixed point E, in the radius BD produced, bisecting the semicircle ABC, so that any line EFG being drawn from it, this line shall always cut the perpendicular radius AD and the quadrantal arc AB, proportionally in the two points F and G; viz. so that DF shall be to BG in a constant ratio.

Solution.—Put the radius AD or DB $= r$, DE $= ar$, the arc BG $= z$, GH $= y$, and DF $= v$. Now, if z to v be a constant ratio, then $\dot{z}$ to $\dot{v}$ will also be constant; and the contrary. But, by similar triangles, EH $= ar + \sqrt{(r^2 - y^2)}$: GH $= y$: : ED $= ar$: DF $= \frac{ary}{ar + \sqrt{(r^2-y^2)}} = v$; the fluxion of which is $ar\dot{y} \times \frac{r^2 + arw}{w(ar+w)^2} = \dot{v}$; putting $w = \sqrt{(r^2 - y^2)}$ = DH; also $\dot{z} = \dot{y} \times \frac{r}{\sqrt{(r^2-y^2)}} = \dot{y} \times \frac{r}{w}$. Hence then $\dot{z} : \dot{v} :: \frac{r}{w} : ar \times \frac{r^2+arw}{w(ar+w)^2} :: 1 : ar \times \frac{r+aw}{(ar+w)^2}$; which is evidently a variable ratio. Therefore there is no such fixed point E, as that mentioned in the problem.

Corollary 1.—Hence then it appears, that the common method of finding the side of a polygon inscribed in a circle, by drawing a line from a certain fixed point E, through F and G, making AF to AC as 2 is to the number of sides of the polygon, is not generally true.

Corol. 2.—But such a point E may be found, as shall render that construction at least *nearly* true, in the following manner. Suppose the line EFG to revolve about E, from B to A: at B, the arc BG and the line DF arise in the ratio of BE to DE; and at A they are in the ratio of BA to AD or DB; therefore make these two ratios equal to each other, and it will determine the point E, so as that the ratios in all the intermediate points, or situations, will be nearly equal: thus then, BE : DE : : BA : AD : : p : 2, making $p = 3{\cdot}1416$; or BD : DE : : $p - 2 : 2$; hence DE $= \frac{2}{p-2} \times$ BD $= 1{\cdot}752$ BD $= \frac{7}{4}$ BD very nearly. If, therefore, DE be taken to DA as 7 to 4; then any line drawn from E, to cut the diameter AC, and the semicircumference ABC, it will very nearly cut them proportionally. Therefore, if a polygon is to be inscribed, or if the whole circumference is to be divided into any number of equal parts; first divide the diameter into the same number of parts, and through the 2d point of division draw EFG, so will AG be one of the equal parts very nearly.

Corol. 3.—The number 1·752 being equal to $\sqrt{3}$ nearly, for $\sqrt{3} = 1\cdot732$; therefore, if DE be taken to DA as $\sqrt{3}$ to 1, the point E will be found answering the same purpose as before, but not quite so near as the former. And here, because DA : DE :: 1 : $\sqrt{3}$, therefore DE is the perpendicular of an equilateral triangle described on AC. Hence then, if with the centres A, C, and radius AC, two arcs be described, they will intersect in the point E, nearly the same as before. And this is the method in common practice; but it is not so near the truth as the construction in the 2d Corollary.

Corol. 4.—Hence also a right line is found equal to the arc of a circle nearly: for BG is $= \frac{11}{7}$ DF nearly. And this is the same as the ratio of 11 to 7, which Archimedes gave for the ratio of the semicircumference to the diameter, or 22 to 7 the ratio of the whole circumference to the diameter. But the proportion is here rendered general for any arc of the circle, as well as for the whole circumference.

TRACT XVI.

ON PLANE TRIGONOMETRY WITHOUT TABLES.

THE cases of trigonometry are usually calculated by means of tables of sines, tangents or secants, either of their natural numbers, or their logarithms. But the calculations may also be made without any such tables, to a tolerable degree of accuracy, by means of the theorems and rules contained in the following propositions and corollaries.

PROPOSITION.

If $2a$ denote a side of any triangle, A the number of degrees contained in its opposite angle, and r the radius of the circle