

## P R E F A C E.

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HAVING been, for a long series of years, in the constant habit of preserving original Tracts and dissertations on scientific subjects; and now enjoying, at a very advanced period of life, some degree of leisure, in consequence of my retirement from the laborious duties of the Royal Military Academy; I have anxiously embraced the opportunity of selecting, and revising, such of those papers as were likely to be most useful, and of presenting them to the public.

Some few parts of these Tracts have been already printed in the Philosophical Transactions, and in other works; but most of them are quite new; and such as are not so, having been recast and greatly improved, may be also considered in some measure as original compositions. These papers, being necessarily of a miscellaneous nature, are here arranged nearly according to the order of time in which they were composed; and the description of them, is briefly as follows.

### VOLUME I.

TRACT I, is on the Principles of Bridges.—The original of this paper was a small pamphlet on the same subject, first published by me on a particular occasion at Newcastle, in the year 1772. It was also republished at London in 1801, nearly in the same state. But it has been now recomposed, and greatly enlarged with many additional propositions, as also numerous observations, both practical and scientific.

An Appendix is also added, containing my report to the Committee of Parliament on the project for a new iron



bridge, of only one arch, proposed to be thrown over the river Thames at London; with several other appropriate articles, as below.

TRACT II, exhibits some curious queries concerning London Bridge, proposed in the year 1746 by the magistrates of the city; with the ingenious answers given to the same, by Mr. George Dance, surveyor-general of the city works, being the result of that gentleman's examination concerning the state of the bridge at that time.

TRACT III contains experiments and observations to be made on the state of London bridge; being the report of a committee of the members of the Royal Society, addressed to the common council of the city of London.

TRACT IV treats of the effects which might be produced on the tides in the river Thames, in consequence of erecting a bridge at Blackfriars. This was an ingenious report, drawn up by the late Mr. John Robertson, at the request of the city of London.

TRACT V consists of answers, given by me, to questions proposed by the Select Committee of Parliament, relative to a proposal, made by Messrs. Telford and Douglas, for erecting a new iron bridge, of a single arch only, over the river Thames, instead of the present London bridge.

TRACT VI exhibits a brief history of the original invention, and subsequent improvements of iron bridges, as practised of late years in this country.

TRACT VII is a dissertation on the nature and value of infinite series; explaining the properties of several forms of such series, as converging, diverging, and neutral.

TRACT VIII is a new method for the valuation of numeral infinite series, that have their terms alternately plus and minus; which is performed by taking continual arithmetical means between the successive sums, and between the means; a method by which the value or sum of any such series is very easily and quickly obtained.



TRACT IX is a method of summing the series  $a+bx+cx^2+dx^3+ex^4+\&c$ , in the case when it converges very slowly, namely, when  $x$  is nearly equal to 1, and the coefficients  $a, b, c, d, \&c$ , decrease very slowly; the signs of all the terms being plus or positive:—a method which has been considered a great desideratum in infinite series.

TRACT X contains the investigation of certain easy and general rules, for extracting any root out of a given number; exhibiting a general and very easy formula, to serve for all roots whatever.

TRACT XI is a new method of finding, in general and finite terms, near values of the roots of equations of this form,  $x^n - px^{n-1} + qx^{n-2} - \&c = 0$ ; namely, having the terms alternately plus and minus: being one method more to be added to the many we are already possessed of, for determining the roots of the higher orders of equations.

TRACT XII treats of the binomial theorem; exhibiting a demonstration of the truth of it in the general case of fractional exponents. The demonstration is of this nature, that it proves the law of the whole series in a formula of one single term only: thus,  $p, q, r$ , denoting any three successive terms of the series, expanded from the given binomial  $(1+x)^{\frac{1}{n}}$ , and if  $\frac{q}{p} = \alpha$ , then is  $\frac{r}{q} = \alpha + \frac{1}{n}$ , which denotes the general law of the series, being a new mode of proving the law of the coefficients of this celebrated theorem. But, besides this law of the coefficients, the very form of the series is, for the first time, here demonstrated, viz, that the form of the series for the developement of the binomial  $(1+x)^{\frac{1}{n}}$ , with respect to the exponents, will be  $1+ax+bx^2+cx^3+dx^4+\&c$ , a form which has heretofore been assumed without proof.

TRACT XIII treats on the common sections of the sphere and cone: with the demonstration of some other new properties of the sphere, which are similar to certain known properties of the circle. The few propositions which form



part of this tract, is a small specimen of the analogy, and even identity, of some of the more remarkable properties of the circle, with those of the sphere. To which are added some properties of the lines of section, and of contact, between the sphere and cone: both of which can be further extended as occasions may offer.

TRACT XIV, on the geometrical division of circles and ellipses into any number of parts having equal perimeters, and areas either all equal or in any proposed ratios to each other: constructions which were never before given by any author, but which, on the contrary, had been accounted impossible to be effected.

TRACT XV contains an approximate geometrical division of the circumference of the circle.

TRACT XVI treats on plane trigonometry, without the use of the common tables of sines, tangents, and secants: resolving all the cases in numbers, by means of certain algebraical formulæ only.

TRACT XVII is on Machin's quadrature of the circle; being an investigation of that learned gentleman's very simple and easy series for that purpose, by help of the tangent of the arc of 45 degrees; which series the author had given without any proof or investigation.

TRACT XVIII, a new and general method of finding simple and quickly-converging series; by which the proportion of the diameter of a circle to its circumference may easily be computed to a great many places of figures. By this method are found, not only Machin's series, noticed in the last Tract, but also several others that are much more simple and easy than his.

TRACT XIX, the history of trigonometrical tables, &c: being a critical description of all the writings on trigonometry made before the invention of logarithms.

TRACT XX, the history of logarithms; giving an account of the inventions and descriptions by several authors on the different kinds of logarithms.



TRACT XXI, on the construction of logarithms; exhibiting the various and peculiar methods employed by all the different authors, in their several computations of these very useful numbers.

TRACT XXII, treats on the powers of numbers; chiefly relating to curious properties of the squares, and the cubes, and other powers of numbers.

TRACT XXIII, is a new and easy method of extracting the square roots of numbers; very useful in practice.

TRACT XXIV, shows how to construct tables of the square-roots, and cube-roots, and the reciprocals of the series of the natural numbers; being a general method, by means of the law of the differences of such roots and reciprocals of numbers.

TRACT XXV, is an extensive table of roots and reciprocals, constructed in the above manner, accompanied also with the series of the squares and cubes of the same numbers.

## VOLUME II.

TRACT XXVI, an account of the calculations made from the survey and measures taken at mount Shichallin, in order to ascertain the mean density of the earth: being the result of a laborious calculation, the first ever made to ascertain that density; by which it is shown to be nearly equal to 5 times the density of water, or almost double the density of the rocks at the surface of the earth, and that consequently the interior of the earth must consist of immense quantities of metals or metallic ores.

TRACT XXVII, consists of calculations to determine at what point, on the side of a hill, its attraction will be the greatest. This is inserted as an appendix to the preceding tract, and intended to direct operations of any future attempt to ascertain such density, or to corroborate the foregoing statement; and, by this determination, it is shown that the best situation is generally at about  $\frac{1}{4}$  of the altitude of the hill.



TRACT XXVIII, is an extensive treatise on cubic equations and infinite series: showing their nature, properties, and solutions, both in finite formulas and by expressions in infinite series.

TRACT XXIX contains a curious project for a new division of the quadrantal arc of the circle, with a view to trigonometrical and other purposes: being intended for the novel design of constructing tables of the sines, tangents, and secants of arcs, to equal parts of the radius of the circle; or expressing all these lines, as well as the arcs themselves, in such parts.

TRACT XXX, on the sections of spheroids and conoids: showing that all such plane sections are the same as conic sections; and that all the parallel sections, in each of these solids, are like and similar figures.

TRACT XXXI, on the comparison of curves of the same species; showing their mutual relations.

TRACT XXXII contains a theorem for the cube-root of an algebraic binomial, one of the terms being a quadratic radical; useful in the solution of certain cubic equations by Cardan's rule.

TRACT XXXIII, is a complete history of algebra; tracing its origin and practice among the ancient Greeks, the Indians, Persians, and Arabians; with particular details of the various peculiarities and improvements, made among different people, and by several eminent individuals, especially among the European authors, namely, the Italians, Spaniards, French, Germans, and the English; in which all the discoveries and improvements are ascribed to the proper authors.

TRACT XXXIV, exhibits the results of new experiments in Artillery, for determining the force of fired gunpowder, the initial velocity of cannon balls, the ranges of projectiles at different elevations, the resistance of the air to their motions; the effect of different lengths of guns, and of different quan-



tities of powder, &c, &c: giving a complete detail of all the circumstances attending these very numerous and accurate experiments, with many useful philosophical and practical inferences deduced from them; the whole forming as it were a new era in the progress of this curious and important branch of knowledge.

## VOLUME III.

TRACT XXXV, on a new Gunpowder Eprouvette; showing its construction and use, by means of which the strength and quality of gunpowder may be proved and evinced, in a way far more exact and easy than by any other machine.

TRACT XXXVI, on the Resistance of the Air to bodies in motion, as determined by the Whirling Machine: showing the exact quantity of the air's resistance to all forms of bodies, moved through it with slow and moderate motions; the effects of which, combined with those of the very high motions of cannon and musket shot, furnish us with a complete and uniform series of resistances to all degrees of velocity, from the very slowest perceptible motions, to those of the highest and most violent.

TRACT XXXVII, on the Theory and Practice of Gunnery, as dependent on the Resistance of the Air. This tract is employed in stating the deductions abstracted from all the preceding experiments, and applying them in many problems, to the important purposes of Artillery and projectiles. Here are given complete tables of the quantity of resistance to balls moving with every degree of velocity; with correct rules for ascertaining those that are proper to all other sizes of balls. Here are also given general rules and algebraic formulæ, for expressing the resistance to any size of ball in terms of the velocity; with a great variety of problems for determining the motions of balls in all directions, upwards, downwards, or obliquely, touching their velocities and times in motion, with the ranges of projectiles in the air,



and practical applications to the cases of gunnery, in a great variety of useful instances.

TRACT XXXVIII, being the last, contains a miscellaneous collection of practical questions, illustrating several of the principles in the preceding Tracts, with the solutions at large.

Such are the outlines of a work, which is the result of many years assiduous study and persevering research; and which it is presumed will be found to contain several new articles, on civil and military science, that may be deemed of national importance.

It is, in all probability, the last original work that I may ever be able to offer to the notice of the Public, and I am therefore the more anxious that it should be found worthy of their acceptance and regard. To their kind indulgence, indeed, is due whatever success I may have experienced, both as an Author and Teacher for more than half a century: and it is no small satisfaction to reflect, that my humble endeavours, during that period, have not been wholly unsuccessful in the diffusion of useful knowledge.

To the same liberal encouragement of the Public must likewise be ascribed, in a great measure, the means of the comfortable retirement which I now enjoy, towards the close of a long and laborious life: and for which I have every reason to be truly thankful.

CHA. HUTTON.

*London,*  
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