Sammlung

der vorzüglichsten

Integralformeln.

I. $X = a + bx$.

$$\int \frac{dx}{X} = \frac{1}{b} \log. X,$$

$$\int \frac{x\,dx}{X} = \frac{x}{b} - \frac{a}{b^2} \log. X,$$

$$\int \frac{x^2\,dx}{X} = \frac{x^2}{2b} - \frac{ax}{b^2} + \frac{a^2}{b^3} \log. X,$$

$$\int \frac{x^m\,dx}{X} = \frac{x^m}{bm} - \frac{a}{b} \int \frac{x^{m-1}\,dx}{X}.$$

$$\int \frac{dx}{X^2} = -\frac{1}{bX},$$

$$\int \frac{dx}{x^n} = -\frac{1}{(n-1)\,b\,X^{n-1}},$$

$$\int \frac{x\,dx}{X^2} = \frac{a}{b^2 X} + \frac{1}{b^2} \log. X,$$

$$\int \frac{x^2\,dx}{X^2} = \left(\frac{x^2}{b} - \frac{2a^2}{b^3}\right) \frac{1}{X} - \frac{2a}{b^3} \log. X,$$

$$\int \frac{x^m\,dx}{X^2} = \frac{x^m}{(m-1)\,bX} - \frac{am}{(m-1)\,b} \int \frac{x^{m-1}\,dx}{X^2}.$$

$$\int \frac{dx}{X^3} = -\frac{1}{2bX^2},$$

$$\int \frac{x\,dx}{X^3} = -\left(\frac{x}{b} + \frac{a}{2b^2}\right) \frac{1}{X^2},$$

$$\int \frac{x^2\,dx}{X^3} = \left(\frac{2ax}{b^2} + \frac{3a^2}{2b^3}\right) \frac{1}{X^2} + \frac{1}{b^3} \log. X.$$

$$\int \frac{dx}{xX} = \frac{1}{a} \log. \frac{x}{X},$$

$$\int \frac{dx}{x^2 X} = -\frac{1}{ax} + \frac{b}{a^2} \log. \frac{X}{x},$$

$$\int \frac{dx}{x^3 X} = -\frac{1}{2ax^2} + \frac{b}{a^2 x} - \frac{b^2}{a^3} \log. \frac{X}{x},$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)\,a\,x^{m-1}} - \frac{b}{a} \int \frac{dx}{x^{m-1} X}.$$

$$\int \frac{dx}{xX^2} = \frac{1}{aX} - \frac{1}{a^2} \log. \frac{X}{x},$$

$$\int \frac{dx}{x^2X^2} = -\left(\frac{1}{ax} + \frac{2b}{a^2}\right)\frac{1}{X} + \frac{2b}{a^3} \log. \frac{X}{x}.$$

$$\int \frac{dx}{\sqrt{X}} = \frac{2}{b}\sqrt{X},$$

$$\int \frac{x\,dx}{\sqrt{X}} = (\tfrac{1}{3}X - a)\,\frac{2\sqrt{X}}{b^2},$$

$$\int \frac{x^2\,dx}{\sqrt{X}} = (\tfrac{1}{5}X^2 - \tfrac{2}{3}aX + a^2)\,\frac{2\sqrt{X}}{b^3},$$

$$\int \frac{x^3\,dx}{\sqrt{X}} = (\tfrac{1}{7}X^3 - \tfrac{3}{5}aX^2 + a^2X - a^3)\,\frac{2\sqrt{X}}{b^4}.$$

$$\int \frac{dx}{x\sqrt{X}} = \frac{1}{\sqrt{a}} \log. \frac{\sqrt{X} - \sqrt{a}}{\sqrt{X} + \sqrt{a}},$$ wenn a positiv,

$$= \frac{2}{\sqrt{-a}} \text{ arc. tang. } \frac{\sqrt{X}}{\sqrt{-a}},$$ wenn a negativ,

$$\int \frac{dx}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a}\int \frac{dx}{x\sqrt{X}},$$

$$\int \frac{dx}{x^3\sqrt{X}} = -\left(\frac{1}{2ax^2} - \frac{3b}{4a^2x}\right)\sqrt{X} + \frac{3b^2}{8a^2}\int \frac{dx}{x\sqrt{X}}.$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = -\frac{2}{b\sqrt{X}},$$

$$\int \frac{x\,dx}{X^{\frac{3}{2}}} = (X + a)\,\frac{2}{b^2\sqrt{X}},$$

$$\int \frac{x^2\,dx}{X^{\frac{3}{2}}} = (\tfrac{1}{3}X^2 - 2aX - a^2)\,\frac{2}{b^3\sqrt{X}},$$

$$\int \frac{dx}{xX^{\frac{3}{2}}} = \frac{2}{a\sqrt{X}} + \frac{1}{a}\int \frac{dx}{x\sqrt{X}},$$

$$\int \frac{dx}{x^2X^{\frac{3}{2}}} = -\left(\frac{1}{ax} + \frac{3b}{a^2}\right)\frac{1}{\sqrt{X}} - \frac{3b}{2a^2}\int \frac{dx}{x\sqrt{X}}.$$

$$\int dx\sqrt{X} = \frac{2X\sqrt{X}}{3b},$$

$$\int x\,dx\sqrt{X} = (\tfrac{1}{5}X - \tfrac{1}{3}a)\,\frac{2X\sqrt{X}}{b^2},$$

$$\int x^2\,dx\sqrt{X} = (\tfrac{1}{7}X^2 - \tfrac{2}{5}aX + \tfrac{1}{3}a^2)\,\frac{2X\sqrt{X}}{b^3}.$$

$$\int \frac{dx\sqrt{X}}{x} = 2\sqrt{X} + a\int \frac{dx}{x\sqrt{X}},$$

$$\int \frac{dx\sqrt{X}}{x^2} = -\frac{\sqrt{X}}{x} + \frac{b}{2}\int \frac{dx}{x\sqrt{X}},$$

$$\int \frac{dx\sqrt{X}}{x^3} = -\frac{X\sqrt{X}}{2ax^2} + \frac{b\sqrt{X}}{4ax} - \frac{b^2}{8a}\int \frac{dx}{x\sqrt{X}}.$$

$$\int dx\,.\,X^{\frac{3}{2}} = \frac{2X^2\sqrt{X}}{5b},$$

$$\int x\,dx\,.\,X^{\frac{3}{2}} = (\tfrac{1}{7}X - \tfrac{1}{5}a)\,\frac{2X^2\sqrt{X}}{b^2},$$

$$\int x^2\,dx\,.\,X^{\frac{3}{2}} = (\tfrac{1}{9}X^2 - \tfrac{2}{7}aX + \tfrac{1}{5}a^2)\,\frac{2X^2\sqrt{X}}{b^3},$$

$$\int \frac{dx\,.\,X^{\frac{3}{2}}}{x} = (\tfrac{1}{3}X + a)\,2\sqrt{X} + a^2\int \frac{dx}{x\sqrt{X}},$$

$$\int \frac{dx\,.\,X^{\frac{3}{2}}}{x^2} = -\frac{X^2\,dx}{ax} + \frac{3b}{2a}\int \frac{dx\,.\,X^{\frac{3}{2}}}{x},$$

$$\int \frac{dx}{\sqrt[3]{X}} = \frac{3\sqrt[3]{X^2}}{2b},$$

$$\int \frac{x\,dx}{\sqrt[3]{X}} = (\tfrac{1}{5}X - \tfrac{1}{2}a)\,\frac{3\sqrt[3]{X^2}}{b^2},$$

$$\int \frac{dx}{x\sqrt[3]{X}} = \frac{1}{\sqrt[3]{a}}\left[\tfrac{3}{2}\log.\frac{\sqrt[3]{X} - \sqrt[3]{a}}{\sqrt[3]{x}} + \sqrt{3}\,.\,\text{arc. tang.}\frac{\sqrt{3}\,.\,\sqrt[3]{X}}{\sqrt[3]{X} + 2\sqrt[3]{a}}\right],$$

$$\int dx\,.\,\sqrt[3]{X} = \frac{3X\sqrt[3]{X}}{4b},$$

$$\int dx\,.\,\sqrt[3]{X^2} = \frac{3X\sqrt[3]{X}}{5b}.$$

$$\int \frac{dx}{X\sqrt{x}} = \pm\frac{2}{\sqrt{ab}}\ \text{arc. tang.}\sqrt{\frac{bx}{a}},$$ wenn a und b gleiche Zeichen haben;

$$= \frac{1}{\sqrt{(-ab)}}\log.\frac{a - bx + 2\sqrt{x}\,.\,\sqrt{(-ab)}}{X},$$ wenn a und b ungleiche Zeichen haben;

$$\int \frac{dx}{X^2\sqrt{x}} = \frac{\sqrt{x}}{aX} + \frac{1}{2a}\int \frac{dx}{X\sqrt{x}},$$

$$\int \frac{dx\sqrt{x}}{X} = \frac{2\sqrt{x}}{b} - \frac{a}{b}\int \frac{dx}{X\sqrt{x}},$$

$$\int \frac{x\,dx\,.\sqrt{x}}{X} = \left(\frac{x}{3b} - \frac{a}{b^2}\right) 2\sqrt{x} + \frac{a^2}{b^2}\int\frac{dx}{X\sqrt{x}},$$

$$\int \frac{dx\,.\sqrt{x}}{X^2} = -\frac{\sqrt{x}}{bX} + \frac{1}{2b}\int\frac{dx}{X\sqrt{x}},$$

$$\int \frac{x\,dx\,.\sqrt{x}}{X^2} = \frac{2x\sqrt{x}}{bX} - \frac{3a}{b}\int\frac{dx\sqrt{x}}{X^2},$$

$$\int \frac{dx}{X\sqrt{x}} = \frac{1}{bk^3\sqrt{2}}\left[\log.\frac{x + k\sqrt{2x} + k^2}{\sqrt{X}} + \text{arc. tang.}\frac{k\sqrt{2x}}{k^2 - x}\right],$$

wenn a und b dieselben Zeichen haben, wo $k = \sqrt[4]{\frac{a}{b}}$ ist;

$$\int \frac{dx}{X\sqrt{x}} = \frac{1}{2bk^3}\left[\log.\frac{k - \sqrt{x}}{k + \sqrt{x}} - 2\ \text{arc. tang.}\frac{\sqrt{x}}{k}\right],$$

wenn a und b verschiedene Zeichen haben, wo $k = \sqrt[4]{-\frac{a}{b}}$ ist;

$$\int \frac{dx}{X^2\sqrt{x}} = \frac{\sqrt{x}}{2aX} + \frac{3}{4a}\int\frac{dx}{X\sqrt{x}},$$

$$\int \frac{dx\sqrt{x}}{X} = \frac{1}{bk\sqrt{2}}\left[\text{arc. tang.}\frac{k\sqrt{2x}}{k^2 - x} - \log.\frac{x + k\sqrt{2x} + k^2}{\sqrt{X}}\right],$$

wenn a und b dieselben Zeichen haben, wo $k = \sqrt[4]{\frac{a}{b}}$ ist;

$$\int \frac{dx\sqrt{x}}{X} = \frac{1}{2bx}\left[\log.\frac{k - \sqrt{x}}{k + \sqrt{x}} + 2\ \text{arc. tang.}\frac{\sqrt{x}}{k}\right],$$

wenn a und b verschiedene Zeichen haben, wo $k = \sqrt[4]{-\frac{a}{b}}$ ist;

$$\int \frac{x\,dx\sqrt{x}}{X} = \frac{2\sqrt{x}}{b} - \frac{a}{b}\int\frac{dx}{X\sqrt{x}},$$

$$\int \frac{dx\sqrt{x}}{X^2} = \frac{x\sqrt{x}}{2aX} + \frac{1}{4a}\int\frac{dx\sqrt{x}}{X},$$

$$\int \frac{x\,dx\,.\sqrt{x}}{X^2} = -\frac{\sqrt{x}}{2bX} + \frac{1}{4b}\int\frac{dx}{X\sqrt{x}},$$

$$\int \frac{dx}{Xx\sqrt{x}} = -\frac{2}{a\sqrt{x}} - \frac{b}{a}\int\frac{dx}{X\sqrt{x}},$$

$$\int \frac{dx}{X^2x\sqrt{x}} = -\frac{2}{aX\sqrt{x}} - \frac{3b}{a}\int\frac{dx}{X^2\sqrt{x}}.$$

II. $X = a + bx^2$.

$$\int\frac{dx}{X} = \frac{1}{\sqrt{ab}}\ \text{arc. tang. } x\sqrt{\frac{b}{a}},\ \text{wenn b positiv,}$$

$$= \frac{1}{2\sqrt{-ab}}\ \log.\frac{\sqrt{a} + x\sqrt{-b}}{\sqrt{a} - x\sqrt{-b}},\ \text{wenn b negativ;}$$

$$\int\frac{dx}{X^2} = \frac{x}{2aX} + \frac{1}{2a}\int\frac{dx}{X},$$

$$\int\frac{dx}{X^3} = \left(\frac{1}{4aX^2} + \frac{3}{8a^2X}\right)x + \frac{3}{8a^2}\int\frac{dx}{X},$$

$$\int\frac{x\,dx}{X} = \frac{1}{2b}\log. X,$$

$$\int\frac{x^2\,dx}{X} = \frac{x}{b} - \frac{a}{b}\int\frac{dx}{X},$$

$$\int\frac{x^3\,dx}{X} = \frac{x^2}{2b} - \frac{a}{b}\int\frac{x\,dx}{X},$$

$$\int\frac{x\,dx}{X^2} = -\frac{1}{2bX},$$

$$\int\frac{x^2\,dx}{X^2} = -\frac{x}{2bX} + \frac{1}{2b}\int\frac{dx}{X},$$

$$\int\frac{x^3\,dx}{X^2} = \frac{a}{2b^2X} + \frac{1}{2b^2}\log. X,$$

$$\int\frac{dx}{X^3} = \left(\frac{3bx^3}{8a^2} + \frac{5x}{8a}\right)\frac{1}{X^2} + \frac{3}{8a^2}\int\frac{dx}{X},$$

$$\int\frac{dx}{xX} = \frac{1}{2a}\log.\frac{x^2}{X},$$

$$\int\frac{dx}{x^2X} = -\frac{1}{ax} - \frac{b}{a}\int\frac{dx}{X},$$

$$\int\frac{dx}{xX^2} = \frac{1}{2aX} + \frac{1}{a}\int\frac{dx}{xX},$$

$$\int\frac{dx}{x^2X^2} = -\left(\frac{1}{ax} + \frac{3bx}{2a^2}\right)\frac{1}{X} - \frac{3b}{2a^2}\int\frac{dx}{X},$$

$$\int\frac{dx}{X^m} = \frac{x}{2(m-1)aX^{m-1}} + \frac{2m-3}{2a(m-1)}\int\frac{dx}{X^{m-1}},$$

$$\int\frac{x^m\,dx}{X} = \frac{x^{m-1}}{b(m-1)} - \frac{a}{b}\int\frac{x^{m-2}\,dx}{X},$$

$$\int\frac{x^m\,dx}{X^2} = \frac{x^{m-1}}{b(m-3)X} - \frac{a(m-1)}{b(m-3)}\int\frac{x^{m-2}\,dx}{X^2},$$

$$\int\frac{dx}{x^mX} = -\frac{1}{a(m-1)x^{m-1}} - \frac{b}{a}\int\frac{dx}{x^{m-2}X},$$

$$\int\frac{dx}{x^mX^2} = -\frac{1}{a(m-1)x^{m-1}X} - \frac{b(m+1)}{a(m-1)}\int\frac{dx}{x^{m-2}X^2},$$

$$\int \frac{dx}{\sqrt{X}} = \frac{1}{\sqrt{b}} \log. [x\sqrt{b} + \sqrt{X}], \text{ wenn } b \text{ positiv,}$$

$$= \frac{1}{\sqrt{-b}} \text{ arc. sin. } x \sqrt{-\frac{b}{a}}, \text{ wenn } b \text{ negativ;}$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = \frac{x}{a\sqrt{X}},$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = \left(\frac{1}{3aX} + \frac{2}{3a^2}\right) \frac{x}{\sqrt{X}},$$

$$\int \frac{x\,dx}{\sqrt{X}} = \frac{\sqrt{X}}{b},$$

$$\int \frac{x^2\,dx}{\sqrt{X}} = \frac{x\sqrt{X}}{2b} - \frac{a}{2b} \int \frac{dx}{\sqrt{X}},$$

$$\int \frac{dx}{x\sqrt{X}} = \frac{1}{2\sqrt{a}} \log. \frac{\sqrt{X} - \sqrt{a}}{\sqrt{X} + \sqrt{a}}, \text{ wenn } a \text{ positiv,}$$

$$= \frac{1}{\sqrt{-a}} \text{ . arc. sec. } x \sqrt{-\frac{b}{a}}, \text{ wenn } a \text{ negativ;}$$

$$\int \frac{dx}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{ax},$$

$$\int \frac{dx}{x^3\sqrt{X}} = -\frac{\sqrt{X}}{2ax^2} - \frac{b}{2a} \int \frac{dx}{x\sqrt{X}},$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = \frac{x}{a\sqrt{X}},$$

$$\int \frac{x\,dx}{X^{\frac{3}{2}}} = -\frac{1}{b\sqrt{X}},$$

$$\int \frac{dx}{xX^{\frac{3}{2}}} = \frac{1}{a\sqrt{X}} + \frac{1}{a} \int \frac{dx}{x\sqrt{X}},$$

$$\int \frac{x^2\,dx}{X^{\frac{3}{2}}} = -\frac{x}{b\sqrt{X}} + \frac{1}{b} \int \frac{dx}{\sqrt{X}},$$

$$\int \frac{dx}{x^2 . X^{\frac{3}{2}}} = -\left(\frac{1}{ax} + \frac{2bx}{a^2}\right) \frac{1}{\sqrt{X}},$$

$$\int dx\sqrt{X} = \tfrac{1}{2} x\sqrt{X} + \frac{a}{2\sqrt{b}} \log. (x\sqrt{b} + \sqrt{X}), \text{ wenn } b \text{ positiv,}$$

$$= \tfrac{1}{2} x\sqrt{X} + \frac{a}{2\sqrt{-b}} \text{arc. sin. } x \sqrt{-\frac{b}{a}}, \text{ wenn } b \text{ negativ;}$$

$$\int x\,dx\sqrt{X} = \frac{X\sqrt{X}}{3b},$$

$$\int x^2\,dx\sqrt{X} = \frac{xX\sqrt{x}}{4b} - \frac{a}{4b} \int dx\sqrt{X},$$

$$\int x^3 dx \sqrt{X} = \left(\frac{x^2}{5b} - \frac{2a}{15b^2}\right) X\sqrt{X},$$

$$\int \frac{dx\sqrt{X}}{x} = \sqrt{X} + a\int\frac{dx}{x\sqrt{X}},$$

$$\int \frac{dx\sqrt{X}}{x^2} = -\frac{\sqrt{X}}{x} + b\int\frac{dx}{\sqrt{X}},$$

$$\int \frac{dx\sqrt{X}}{x^3} = -\frac{\sqrt{X}}{2x^2} + \frac{b}{2}\int\frac{dx}{x\sqrt{X}},$$

$$\int dx\,.\,X^{\frac{3}{2}} = \left(\frac{X}{4} + \frac{3a}{8}\right) x\sqrt{X} + \frac{3a^2}{8}\int\frac{dx}{\sqrt{X}},$$

$$\int x\,dx\,.\,X^{\frac{3}{2}} = \frac{X^2.\sqrt{X}}{5b},$$

$$\int x^2 dx\,.\,X^{\frac{3}{2}} = \frac{xX^2\sqrt{X}}{6b} - \frac{a}{6b}\int dx\,.\,X^{\frac{3}{2}},$$

$$\int \frac{dx\,.\,X^{\frac{3}{2}}}{x} = \left(\frac{X}{3} + a\right)\sqrt{X} + a^2\int\frac{dx}{x\sqrt{X}},$$

$$\int \frac{dx\,.\,X^{\frac{3}{2}}}{x^2} = -\frac{X^2\sqrt{X}}{ax} + \frac{4b}{a}\int dx\,.\,X^{\frac{3}{2}},$$

$$\int \frac{dx}{X\sqrt{x}} = \frac{1}{bk^3\sqrt{2}}\left[\log.\frac{x + k\sqrt{2x} + k^2}{\sqrt{X}} + \text{arc. tang.}\frac{k\sqrt{2x}}{k^2 - x}\right],$$

wenn a und b gleiche Zeichen haben, und $k = \sqrt[4]{\frac{a}{b}}$ ist;

$$\int \frac{dx}{X\sqrt{x}} = \frac{1}{2bk^3}\left[\log.\frac{k - \sqrt{x}}{k + \sqrt{x}} - 2\ \text{arc. tang.}\frac{\sqrt{x}}{k}\right],$$

wenn a und b verschiedene Zeichen haben, und $k = \sqrt[4]{-\frac{a}{b}}$ ist;

$$\int \frac{dx\sqrt{x}}{X} = \frac{1}{bk\sqrt{2}}\left[\text{arc. tang.}\frac{k\sqrt{2x}}{k^2 - x} - \log.\frac{x + k\sqrt{2x} + k^2}{\sqrt{X}}\right],$$

wenn a und b dieselben Zeichen haben, und $k = \sqrt[4]{\frac{a}{b}}$ ist;

$$\int \frac{dx\sqrt{x}}{X} = \frac{1}{2bk}\left[\log.\frac{k - \sqrt{x}}{k + \sqrt{x}} + 2\ \text{arc. tang.}\frac{\sqrt{x}}{k}\right],$$

wenn a und b verschiedene Zeichen haben, und $k = \sqrt[4]{-\frac{a}{b}}$ ist;

$$\int \frac{dx}{X^2\sqrt{x}} = \frac{\sqrt{x}}{2aX} + \frac{3}{4a}\int\frac{dx}{X\sqrt{x}},$$

$$\int \frac{dx}{X^3\sqrt{x}} = \left(\frac{1}{4aX^2} + \frac{7}{16a^2X}\right)\sqrt{x} + \frac{21}{32a^2}\int\frac{dx}{X\sqrt{x}},$$

$$\int \frac{x\,dx\,.\,\sqrt{x}}{X} = \frac{2\sqrt{x}}{b} - \frac{a}{b}\int\frac{dx}{X\sqrt{x}},$$

$$\int \frac{dx \,.\, \sqrt{x}}{X^2} = \frac{x\sqrt{x}}{2aX} + \frac{1}{4a}\int \frac{dx \,.\, \sqrt{x}}{X},$$

$$\int \frac{x\,dx \,.\, \sqrt{x}}{X^2} = -\frac{\sqrt{x}}{2bX} + \frac{1}{4b}\int \frac{dx}{X\sqrt{x}}.$$

III. $X = ax + bx^2$.

$$\int \frac{dx}{\sqrt{X}} = \frac{1}{\sqrt{b}} \log. \frac{\sqrt{X} + x\sqrt{b}}{\sqrt{X} - x\sqrt{b}}, \text{ wenn b positiv,}$$

$$= \frac{2}{\sqrt{-b}} \text{ arc. tang. } \frac{x\sqrt{-b}}{\sqrt{X}}, \text{ wenn b negativ;}$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = -\frac{2(2bx + a)}{a^2\sqrt{X}},$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = -\left(\frac{1}{3X} - \frac{8b}{3a^2}\right)\frac{2(a + 2bx)}{a^2\sqrt{X}},$$

$$\int \frac{x\,dx}{\sqrt{X}} = \frac{\sqrt{X}}{b} - \frac{a}{2b}\int \frac{dx}{\sqrt{X}},$$

$$\int \frac{x^2\,dx}{\sqrt{X}} = \left(\frac{x}{2b} - \frac{3a}{4b^2}\right)\sqrt{X} + \frac{3a^2}{8b^2}\int \frac{dx}{\sqrt{X}},$$

$$\int \frac{x^3\,dx}{\sqrt{X}} = \left(\frac{x^2}{3b} - \frac{5ax}{12b^2} + \frac{5a^2}{8b^3}\right)\sqrt{X} - \frac{5a^3}{16b^3}\int \frac{dx}{\sqrt{X}},$$

$$\int \frac{dx}{x\sqrt{X}} = -\frac{2\sqrt{X}}{ax},$$

$$\int \frac{dx}{x^2\sqrt{X}} = -\left(\frac{1}{3ax^2} - \frac{2b}{3a^2x}\right)2\sqrt{X},$$

$$\int \frac{dx}{x^3\sqrt{X}} = -\left(\frac{1}{5ax^3} - \frac{4b}{15a^2x^2} + \frac{8b^2}{15a^3x}\right)2\sqrt{X},$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = -\frac{2(a + 2bx)}{a^2\sqrt{X}},$$

$$\int \frac{x\,dx}{X^{\frac{3}{2}}} = \frac{2x}{a\sqrt{X}},$$

$$\int \frac{x^2\,dx}{X^{\frac{3}{2}}} = -\frac{2x}{b\sqrt{X}} + \frac{1}{b}\int \frac{dx}{\sqrt{X}},$$

$$\int \frac{dx}{xX^{\frac{3}{2}}} = -\frac{2}{3ax\sqrt{X}} - \frac{4b}{3a}\int \frac{dx}{X^{\frac{3}{2}}},$$

$$\int \frac{dx}{x^2X^{\frac{3}{2}}} = -\left(\frac{1}{5ax^2} - \frac{2b}{5a^2x}\right)\frac{2}{\sqrt{X}} + \frac{8b^2}{5a^2}\int \frac{dx}{X^{\frac{3}{2}}},$$

$$\int dx\sqrt{X} = \left(\frac{x}{2} + \frac{a}{4b}\right)\sqrt{X} - \frac{a^2}{8b}\int\frac{dx}{\sqrt{X}},$$

$$\int x\,dx\sqrt{X} = \frac{X\sqrt{X}}{3b} - \frac{a}{2b}\int dx\sqrt{X},$$

$$\int x^2\,dx\sqrt{X} = \left(\frac{x}{4b} - \frac{5a}{24b^2}\right)X\sqrt{X} + \frac{5a^2}{16b^2}\int dx\sqrt{X},$$

$$\int\frac{dx\sqrt{X}}{x} = \sqrt{X} + \frac{a}{2}\int\frac{dx}{\sqrt{X}},$$

$$\int\frac{dx\sqrt{X}}{x^2} = -\frac{2\sqrt{X}}{x} + b\int\frac{dx}{\sqrt{X}},$$

$$\int dx\,X^{\frac{3}{2}} = \left(\frac{X}{b} - \frac{3a^2}{8b^2}\right)\frac{a+2bx}{8}\sqrt{X} + \frac{3a^4}{128b^2}\int\frac{dx}{\sqrt{X}},$$

$$\int x\,dx\,X^{\frac{3}{2}} = \frac{X^2\sqrt{X}}{5b} - \frac{a}{2b}\int dx\,X^{\frac{3}{2}},$$

$$\int x^2\,dx\,X^{\frac{3}{2}} = \left(\frac{x}{6b} - \frac{7a}{60b^2}\right)X^2\sqrt{X} + \frac{7a^2}{24b^2}\int dx\,X^{\frac{3}{2}},$$

$$\int\frac{dx\,X^{\frac{3}{2}}}{x} = \frac{X\sqrt{X}}{3} + \frac{a}{2}\int dx\sqrt{X},$$

$$\int\frac{dx\,X^{\frac{3}{2}}}{x^2} = \frac{X\sqrt{X}}{2x} + \frac{3a}{4}\sqrt{X} + \frac{3a^2}{8}\int\frac{dx}{\sqrt{X}}.$$

IV. $X = a + bx + cx^2$ und $k = 4ac - b^2$.

$$\int\frac{dx}{X} = \frac{2}{\sqrt{k}}\text{ arc. tang.}\frac{2cx+b}{\sqrt{k}},\ \text{wenn } k \text{ positiv},$$

$$= \frac{1}{\sqrt{-k}}\log.\frac{2cx+b-\sqrt{-k}}{2cx+b+\sqrt{-k}},\ \text{wenn } k \text{ negativ};$$

$$\int\frac{dx}{X^2} = \frac{2cx+b}{kX} + \frac{2c}{k}\int\frac{dx}{X},$$

$$\int\frac{dx}{X^3} = \left(\frac{1}{2kX^2} + \frac{3c}{k^2X}\right)(b+2cx) + \frac{6c^2}{k^2}\int\frac{dx}{X},$$

$$\int\frac{x\,dx}{X} = \frac{1}{2c}\log.X - \frac{b}{2c}\int\frac{dx}{X},$$

$$\int\frac{x^2\,dx}{X} = \frac{x}{c} - \frac{b}{2c^2}\log.X - \left(\frac{a}{c} - \frac{b^2}{2c^2}\right)\int\frac{dx}{X},$$

$$\int\frac{x\,dx}{X^2} = -\frac{1}{2cX} - \frac{b}{2c}\int\frac{dx}{X^2},$$

$$\int\frac{x^2\,dx}{X^2} = -\frac{x}{cX} + \frac{a}{c}\int\frac{dx}{X^2},$$

$$\int\frac{x\,dx}{X^3} = -\frac{1}{4cX^2} - \frac{b}{2c}\int\frac{dx}{X^3}.$$

$$\int \frac{dx}{xX} = \frac{1}{2a} \log. \frac{x^2}{X} - \frac{b}{2a} \int \frac{dx}{X},$$

$$\int \frac{dx}{x^2 X} = -\frac{1}{ax} - \frac{b}{2a^2} \log. \frac{x^2}{X} - \left(\frac{c}{a} - \frac{b^2}{2a^2}\right) \int \frac{dx}{X},$$

$$\int \frac{dx}{xX^2} = \frac{1}{2aX} + \frac{1}{2a^2} \log. \frac{x^2}{X} - \frac{b}{2a} \int \frac{dx}{X^2} - \frac{b}{2a^2} \int \frac{dx}{X},$$

$$\int \frac{dx}{\sqrt{X}} = \frac{1}{\sqrt{c}} \log. \left[b + 2cx + 2c^{\frac{1}{2}} \sqrt{X}\right],$$ wenn c positiv ist,

$$= -\frac{1}{\sqrt{-c}} \text{arc. sin.} \frac{b + 2cx}{\sqrt{b^2 - 4ac}},$$ wenn c negativ ist,

$$\int \frac{dx}{X^{\frac{3}{2}}} = \frac{2(b + 2cx)}{k\sqrt{X}},$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = 2\left(\frac{1}{3kX} + \frac{8c}{3k^2}\right) \frac{(b + 2cx)}{\sqrt{X}},$$

$$\int dx \sqrt{X} = \frac{(b + 2cx)\sqrt{X}}{4c} + \frac{k}{8c} \int \frac{dx}{\sqrt{X}},$$

$$\int dx \,.\, X^{\frac{3}{2}} = \left(\frac{X}{8c} + \frac{3k}{64c^2}\right)(b + 2cx)\sqrt{X} + \frac{3k^2}{128c^2} \int \frac{dx}{\sqrt{X}},$$

$$\int \frac{x\,dx}{\sqrt{X}} = \frac{\sqrt{X}}{c} - \frac{b}{2c} \int \frac{dx}{\sqrt{X}},$$

$$\int \frac{x^2 dx}{\sqrt{X}} = \left(\frac{x}{2c} - \frac{3b}{4c^2}\right)\sqrt{X} + \left(\frac{3b^2}{8c^2} - \frac{a}{2c}\right) \int \frac{dx}{\sqrt{X}},$$

$$\int \frac{dx}{x\sqrt{X}} = \frac{1}{\sqrt{a}} \log. \frac{2a + bx - 2a^{\frac{1}{2}} \,.\, \sqrt{X}}{x},$$ wenn a positiv ist,

$$= -\frac{1}{\sqrt{-a}} \text{arc. tang.} \frac{2a + bx}{2\sqrt{-a} \,.\, \sqrt{X}},$$ wenn a negativ ist,

$$\int \frac{dx}{x^2 \sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a} \int \frac{dx}{x\sqrt{X}},$$

$$\int \frac{dx}{x^3 \sqrt{X}} = -\left(\frac{1}{2ax^2} - \frac{3b}{4a^2 x}\right)\sqrt{X} + \left(\frac{3b^2}{8a^2} - \frac{c}{2a}\right) \int \frac{dx}{x\sqrt{X}},$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = \frac{2(b + 2cx)}{k\sqrt{X}},$$

$$\int \frac{x\,dx}{X^{\frac{3}{2}}} = -\frac{2(2a + bx)}{k\sqrt{X}},$$

$$\int \frac{x^2 dx}{X^{\frac{3}{2}}} = -\frac{(4ac - 2b^2)x - 2ab}{ck \,.\, \sqrt{X}} + \frac{1}{c} \int \frac{dx}{\sqrt{X}},$$

$$\int \frac{dx}{xX^{\frac{3}{2}}} = \frac{1}{a\sqrt{X}} - \frac{b}{2a} \int \frac{dx}{X^{\frac{3}{2}}} + \frac{1}{a} \int \frac{dx}{x\sqrt{X}},$$

$$\int \frac{dx}{x^2 X^{\frac{3}{2}}} = -\left(\frac{1}{ax} + \frac{3b}{2a^2}\right)\frac{1}{\sqrt{X}}$$
$$- \left(\frac{2c}{a} - \frac{3b^2}{4a^2}\right)\int \frac{dx}{X^{\frac{3}{2}}} - \frac{3b}{2a^2}\int \frac{dx}{x\sqrt{X}},$$

$$\int x\,dx\,.\,\sqrt{X} = \frac{X\sqrt{X}}{3c} - \frac{b}{2c}\int dx\sqrt{X},$$

$$\int x^2\,dx\,.\,\sqrt{X} = \left(\frac{x}{4c} - \frac{5b}{24c^2}\right) X\sqrt{X} - \left(\frac{a}{4c} - \frac{5b^2}{16c^2}\right)\int dx\sqrt{X},$$

$$\int \frac{dx\sqrt{X}}{x} = \sqrt{X} + a\int \frac{dx}{x\sqrt{X}} + \frac{b}{2}\int \frac{dx}{\sqrt{X}},$$

$$\int \frac{dx\sqrt{X}}{x^2} = -\frac{\sqrt{X}}{x} + \frac{b}{2}\int \frac{dx}{x\sqrt{X}} + c\int \frac{dx}{\sqrt{X}},$$

$$\int x\,dx\,.\,X^{\frac{3}{2}} = \frac{X^2\sqrt{X}}{5c} - \frac{b}{2c}\int dx\,.\,X^{\frac{3}{2}},$$

$$\int \frac{dx\,.\,X^{\frac{3}{2}}}{x} = \left(a + \frac{X}{3}\right)\sqrt{X} + a^2\int \frac{dx}{x\sqrt{X}} + \frac{ab}{2}\int \frac{dx}{\sqrt{X}}$$
$$+ \frac{b}{2}\int dx\,.\,\sqrt{X},$$

$$\int \frac{dx\,.\,X^{\frac{3}{2}}}{x^2} = -\frac{X^2\sqrt{X}}{ax} + \frac{3b}{2a}\int \frac{dx\,.\,X^{\frac{3}{2}}}{x} + \frac{4c}{a}\int dx\,.\,X^{\frac{3}{2}}.$$

V. Produkte binomischer Faktoren.

$$\int \frac{dx}{(x+a)(x+b)} = \frac{1}{b-a}\log.\frac{x+a}{x+b},$$

$$\int \frac{x\,dx}{(x+a)(x+b)} = \frac{1}{b-a}\left[b\log.(x+b) - a\log.(x+a)\right],$$

$$\int \frac{dx}{(x+a)(x+b)^2} = \frac{1}{(b-a)(x+b)} + \frac{1}{(b-a)^2}\log.\frac{x+a}{x+b},$$

$$\int \frac{x\,dx}{(x+a)(x+b)^2} = -\frac{b}{(b-a)(x+b)} - \frac{a}{(b-a)^2}\log.\frac{x+a}{x+b},$$

$$\int \frac{dx}{(x+a)^2(x+b)^2} = -\frac{1}{(b-a)^2}\left[\frac{1}{x+a} + \frac{1}{x+b}\right]$$
$$- \frac{2}{(b-a)^3}\log.\frac{x+a}{x+b},$$

$$\int \frac{x\,dx}{(x+a)^2(x+b)^2} = \frac{1}{(b-a)^2}\left[\frac{a}{x+a} + \frac{b}{x+b}\right]$$
$$+ \frac{a+b}{(b-a)^3}\log.\frac{x+a}{x+b},$$

$$\int \frac{dx}{(x+a)(x+b)(x+c)} = \frac{1}{(b-a)(c-a)} \log.(x+a),$$
$$+ \frac{1}{(a-b)(c-b)} \log.(x+b),$$
$$+ \frac{1}{(a-c)(b-c)} \log.(x+c),$$

$$\int \frac{x\,dx}{(x+a)(x+b)(x+c)} = \frac{a}{(b-a)(c-b)} \log.(x+a),$$
$$- \frac{b}{(a-b)(c-b)} \log.(x+b),$$
$$- \frac{c}{(a-c)(b-c)} \log.(x+c),$$

$$\int \frac{dx}{(x+a)(x^2+b)} = \frac{1}{a^2+b}\left[\log.\frac{x+a}{\sqrt{x^2+b}} + a\int\frac{dx}{x^2+b}\right],$$

$$\int \frac{x\,dx}{(x+a)(x^2+b)} = \frac{1}{a^2+b}\left[a\log.\frac{\sqrt{x^2+b}}{x+a} + a\int\frac{dx}{x^2+b}\right],$$

$$\int \frac{dx}{(x^2+a)(x^2+b)} = \frac{1}{b-a}\left[\int\frac{dx}{x^2+a} - \int\frac{dx}{x^2+b}\right],$$

$$\int \frac{x\,dx}{(x^2+a)(x^2+b)} = \frac{1}{2(b-a)} \log.\frac{x^2+a}{x^2+b},$$

$$\int \frac{x^2\,dx}{(x^2+a)(x^2+b)} = \frac{1}{a-b}\left[a\int\frac{dx}{x^2+a} - b\int\frac{dx}{x^2+b}\right],$$

$$\int \frac{x\,dx}{(x^2+a)(x+b)^2} = \frac{1}{(a+b)^2}\left[\frac{a-b^2}{2}\log.\frac{(b+x)^2}{x+a}\right.$$
$$\left. + 2ab\int\frac{dx}{x^2+a} + \frac{b}{(a+b^2)(x+b)}\right],$$

$$\int \frac{dx}{(x^2+ax+b)(x+c)} = \frac{1}{c^2-ac+b}\left[\tfrac{1}{2}\log.\frac{(x+c)^2}{x^2+ax+b}\right.$$
$$\left. + (c-\tfrac{1}{2}a)\int\frac{dx}{x^2+ax+b}\right],$$

$$\int \frac{dx}{(a'+b'x)\sqrt{a+bx}} = \frac{2}{\sqrt{b'k}} \text{ arc. tang. } \sqrt{\frac{b'(a+bx)}{k}},$$

wenn b' und $k = a'b - ab'$ gleiche Zeichen haben,

$$= \frac{1}{\sqrt{-b'k}} \log.\frac{a'b - 2ab' - bb'x + 2\sqrt{-b'k}.\sqrt{a+bx}}{a'+b'x},$$

wenn b' und $k = a'b - ab'$ verschiedene Zeichen haben,

$$\int \frac{dx}{(a'+b'x)^2\sqrt{a+bx}} = \frac{\sqrt{a+bx}}{k\sqrt{a'+b'x}} + \frac{b}{2k}\int\frac{dx}{(a'+b'x)\sqrt{a+bx}},$$

wo $k = a'b - ab'$ ist,

$$\int \frac{x\,dx}{(a' + b'x)\sqrt{a + bx}} = \frac{1}{b'} \int \frac{dx}{\sqrt{a + bx}} - \frac{a'}{b'} \int \frac{dx}{(a' + b'x)\sqrt{a + bx}},$$

$$\int \frac{x\,dx}{(a' + b'x)^2 \sqrt{a + bx}} = \frac{1}{b'} \int \frac{dx}{(a' + b'x)\sqrt{a + bx}}$$

$$- \frac{a'}{b'} \int \frac{dx}{(a' + b'x)^2 \sqrt{a + bx}},$$

$$\int \frac{dx}{(a' + b'x^2)\sqrt{a + bx}} = \frac{1}{\sqrt{a'(a'b - ab')}}$$

$$\log. \frac{a'\sqrt{a + bx^2} + x\sqrt{a'(a'b - ab')}}{\sqrt{a' + b'x^2}},$$

wenn $a'(a'b - ab')$ positiv ist,

$$= \frac{1}{\sqrt{a'(ab' - a'b)}} \text{ arc. tang. } \frac{x\sqrt{a'(ab' - a'b)}}{a'\sqrt{a + bx^2}},$$

wenn $a'(a'b - ab')$ negativ ist.

VI. Ausdrücke, welche die Größen $a + bx^3$ und $a + bx^4$ u. s. enthalten.

$$\int \frac{dx}{a + bx^3} = \frac{1}{3bk^2} \left[\tfrac{1}{2} \log. \frac{(x + k)^2}{x^2 - kx + k^2} + 3^{\frac{1}{2}} . \text{ arc. tang. } \frac{x\sqrt{3}}{2k - x} \right],$$

wo $k = \sqrt[3]{\frac{a}{b}}$,

$$\int \frac{x\,dx}{a + bx^3} = -\frac{1}{3bk} \left[\tfrac{1}{2} \log. \frac{(x + k)^2}{x^2 - kx + k^2} - 3^{\frac{1}{2}} . \text{ arc. tang. } \frac{x\sqrt{3}}{2k - x} \right],$$

wo $k = \sqrt[3]{\frac{a}{b}}$,

$$\int \frac{x^2\,dx}{P} = \frac{1}{3b} \log. P, \text{ wo } P = a + bx^3,$$

$$\int \frac{x^3\,dx}{P} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{P},$$

$$\int \frac{dx}{P^2} = \frac{x}{3aP} + \frac{2}{3a} \int \frac{dx}{P},$$

$$\int \frac{x\,dx}{P^2} = \frac{x^2}{3aX} + \frac{1}{3a} \int \frac{x\,dx}{P},$$

$$\int \frac{x^2\,dx}{P^2} = -\frac{1}{3bP},$$

$$\int\frac{dx}{P^3} = \left(\frac{5bx^4}{18a^2} + \frac{4x}{9a}\right)\frac{1}{P^2} + \frac{5}{9a^2}\int\frac{dx}{P},$$

$$\int\frac{dx}{xP} = \frac{1}{a}\log.x - \frac{1}{3a}\log.P,$$

$$\int\frac{dx}{x^2P} = -\frac{1}{ax} - \frac{b}{a}\int\frac{x\,dx}{P},$$

$$\int\frac{dx}{xP^2} = \frac{1}{3aP} - \frac{1}{3a^2}\log.\frac{P}{x^3}.$$

$$\int\frac{dx}{Q} = \frac{1}{4bk^3\sqrt{2}}\left[\log.\frac{x^2+kx\sqrt{2}+k^2}{x^2-kx\sqrt{2}+k^2} + 2\,\text{arc. tang.}\frac{kx\sqrt{2}}{k^2-x^2}\right],$$

wo $Q = a + bx^4$ und $k = \sqrt[4]{\frac{a}{b}}$,

$$\int\frac{x\,dx}{Q} = \frac{1}{2bk^2}\,\text{arc. tang.}\,x^2\sqrt{\frac{b}{a}},\ \text{wo}\ k = \sqrt[4]{\frac{a}{b}},$$

$$\int\frac{x^2dx}{Q} = \frac{1}{4bk\sqrt{2}}\left[2\,\text{arc. tang.}\frac{kx\sqrt{2}}{k^2-x^2} - \log.\frac{x^2+kx\sqrt{2}+k^2}{x^2-kx\sqrt{2}+k^2}\right],$$

wo $k = \sqrt[4]{\frac{a}{b}}$,

$$\int\frac{x^3dx}{Q} = \frac{1}{4b}\log.Q,$$

$$\int\frac{x^4dx}{Q} = \frac{x}{b} - \frac{a}{b}\int\frac{dx}{Q},$$

$$\int\frac{dx}{Q} = -\frac{1}{4bk^3}\left[\log.\frac{x+k}{x-k} + 2\,\text{arc. tang.}\frac{x}{k}\right],$$

wo $k = \sqrt[4]{-\frac{a}{b}}$,

$$\int\frac{x\,dx}{Q} = -\frac{1}{4bk^2}\log.\frac{x^2+k^2}{x^2-k^2},\ \text{wo}\ k = \sqrt[4]{-\frac{a}{b}},$$

$$\int\frac{x^2dx}{Q} = -\frac{1}{4bk}\left[\log.\frac{x+k}{x-k} - 2\,\text{arc. tang.}\frac{x}{k}\right],$$

wo $k = \sqrt[4]{-\frac{a}{b}}$,

$$\int\frac{dx}{Q^2} = \frac{x}{4aQ} + \frac{3}{4a}\int\frac{dx}{Q},$$

$$\int\frac{x\,dx}{Q^2} = \frac{x^2}{4aQ} + \frac{1}{2a}\int\frac{x\,dx}{Q},$$

$$\int\frac{dx}{xQ} = \frac{1}{a}\log.x - \frac{1}{4a}\log.Q.$$

VII. Trigonometrische Differentialien.

$$\int dx \sin.^m x \cos.^n x$$

$$= \frac{\sin.^{m+1} x \cos.^{n-1} x}{m+1} + \frac{n-1}{m+1} \int dx \sin.^{m+2} x \cos.^{n-2} x,$$

$$= \frac{\sin.^{m+1} x \cdot \cos.^{n-1} x}{m+n} + \frac{n-1}{m+n} \int dx \sin.^m x \cos.^{n-2} x,$$

$$= \frac{\sin.^{m+1} x \cos.^{n+1} x}{m+1} + \frac{m+n+2}{m+1} \int dx \sin.^{m+2} x \cos^n x,$$

$$= -\frac{\sin.^{m+1} x \cos.^{n+1} x}{n+1} + \frac{m+n+2}{n+1} \int dx \sin.^m x \cos.^{n+2} x,$$

$$= -\frac{\sin.^{m-1} x \cos.^{n+1} x}{n+1} + \frac{m-1}{n+1} \int dx \sin^{m-2} x \cdot \cos.^{n+2} x,$$

$$= -\frac{\sin.^{m-1} x \cos.^{n+1} x}{m+n} + \frac{m-1}{m+n} \int dx \sin.^{m-2} x \cos.^n x.$$

wo m und n jede willkürliche Zahl bezeichnet.

$\int dx \sin. x = -\cos. x,$
$\int dx \sin.^2 x = -\frac{1}{4} \sin. 2x + \frac{1}{2} x,$
$\int dx \sin.^3 x = \frac{1}{12} \cos. 3x - \frac{3}{4} \cos. x,$
$\int dx \sin.^4 x = \frac{1}{32} \sin. 3x - \frac{1}{4} \sin. 2x + \frac{3}{8} x.$

$\int dx \cos. x = \sin. x,$
$\int dx \cos.^2 x = \frac{1}{4} \sin. 2x + \frac{1}{2} x,$
$\int dx \cos.^3 x = \frac{1}{12} \sin. 3x + \frac{3}{4} \sin. x,$
$\int dx \cos.^4 x = \frac{1}{32} \sin. 4x + \frac{1}{4} \sin. 2x + \frac{3}{8} x.$

$\int dx \sin.^2 x \cos. x = \frac{1}{3} \sin.^3 x,$
$\int dx \sin.^2 x \cos.^2 x = \frac{1}{8} (x - \frac{1}{4} \sin. 4x),$
$\int dx \sin.^2 x \cos.^3 x = (\frac{1}{5} \cos.^2 x + \frac{2}{15}) \sin.^3 x,$
$\int dx \sin.^3 x \cos.^2 x = (\frac{1}{5} \sin.^4 x - \frac{1}{15} \sin.^2 x - \frac{2}{15}) \cos. x,$
$\int dx \sin.^3 x \cos.^3 x = (\frac{1}{6} \cos.^2 x + \frac{1}{12}) \sin.^4 x,$
$\int dx \sin.^3 x \cos. x = \frac{1}{8} (\frac{1}{4} \cos. 4x - \cos. 2x),$
$\int dx \sin.^4 x \cos. x = \frac{1}{16} (\frac{1}{5} \sin. 5x - \sin. 3x + 2 \sin. x),$
$\int dx \sin.^5 x \cos. x = -\frac{1}{32} (\frac{1}{6} \cos. 6x - \cos. 4x + \frac{5}{3} \cos. 2x).$

$$\int \frac{dx}{\sin. x} = \log. \operatorname{tang}. \frac{x}{2},$$

$$\int \frac{dx}{\sin.^2 x} = -\operatorname{cotang}. x,$$

$$\int\frac{dx}{\sin.^3 x} = -\frac{\cos. x}{2\sin.^2 x} + \tfrac{1}{2}\int\frac{dx}{\sin. x},$$

$$\int\frac{dx}{\cos. x} = \log.\,\mathrm{tang.}\,\frac{90+x}{2},$$

$$\int\frac{dx}{\cos^2 x} = \mathrm{tang.}\,x,$$

$$\int\frac{dx}{\cos.^3 x} = \frac{\sin. x}{2\cos.^2 x} + \tfrac{1}{2}\int\frac{dx}{\cos. x},$$

$$\int\frac{dx\,\sin. x}{\cos. x} = -\log.\cos. x,$$

$$\int\frac{dx\,\sin.^2 x}{\cos. x} = -\sin. x + \int\frac{dx}{\cos. x},$$

$$\int\frac{dx\,\sin.^3 x}{\cos. x} = -\tfrac{1}{2}\sin.^2 x + \int\frac{dx\,\sin. x}{\cos. x},$$

$$\int\frac{dx\,\cos. x}{\sin. x} = \log.\sin. x,$$

$$\int\frac{dx\,\cos.^2 x}{\sin. x} = \cos. x + \int\frac{dx}{\sin. x},$$

$$\int\frac{dx\,\cos.^3 x}{\sin. x} = \tfrac{1}{2}\cos.^2 x + \int\frac{dx\,\cos. x}{\sin. x},$$

$$\int\frac{dx\,\sin. x}{\cos.^2 x} = \frac{1}{\cos. x},$$

$$\int\frac{dx\,\sin.^2 x}{\cos.^2 x} = \mathrm{tang.}\,x - x,$$

$$\int\frac{dx\,\sin.^3 x}{\cos.^2 x} = \cos. x + \frac{1}{\cos. x},$$

$$\int\frac{dx\,\sin. x}{\cos.^3 x} = \frac{1}{2\cos.^2 x},$$

$$\int\frac{dx\,\sin.^2 x}{\cos.^3 x} = \frac{\sin. x}{2\cos.^2 x} - \tfrac{1}{2}\int\frac{dx}{\cos. x},$$

$$\int\frac{dx\,\sin.^3 x}{\cos.^3 x} = \frac{1}{2\cos.^2 x} + \log.\cos. x,$$

$$\int\frac{dx}{\sin. x\,\cos. x} = \log.\,\mathrm{tang.}\,x,$$

$$\int\frac{dx}{\sin. x\,\cos.^2 x} = \frac{1}{\cos. x} + \int\frac{dx}{\sin. x},$$

$$\int\frac{dx}{\sin. x\,\cos.^3 x} = \frac{1}{2\cos.^2 x} + \log.\,\mathrm{tang.}\,x,$$

$$\int\frac{dx}{\sin.^2 x\,\cos.^2 x} = -2\,\mathrm{cotang.}\,2x,$$

$$\int\frac{dx}{\sin.^3 x\,\cos.^3 x} = \tfrac{1}{2}\left(\frac{1}{\cos.^2 x} - 3\right)\frac{1}{\sin. x} + \tfrac{3}{2}\int\frac{dx}{\cos. x},$$

$$\int \frac{dx}{\sin.^3 x \cos.^3 x} = -\frac{2 \cos. 2x}{\sin.^2 2x} + 2 \log. \text{tang}. x,$$

$$\int x\,dx \sin. x = -x \cos. x + \sin. x,$$

$$\int x\,dx \cos. x = x \sin. x + \cos. x,$$

$$\int x^2\,dx \sin. x = -x^2 \cos. x + 2x \sin. x + 2 \cos. x,$$

$$\int x^m\,dx \sin. x = -x^m \cos. x + m x^{m-1} \sin. x + m(m-1) x^{m-2} \cos. x,$$
$$- m(m-1)(m-2) x^{m-3} \sin. x,$$
$$- m(m-1)(m-2)(m-3) x^{m-4} \cos. x + + - -$$

$$\int X\,dx \,.\, \text{arc. sin.}\, x = \text{arc. sin}\, x \,.\, \int X\,dx - \int \frac{dx \int X\,dx}{\sqrt{1-x^2}},$$

wo hier und im Folgenden X eine algebraische Funktion von x bezeichnet.

$$\int X\,dx \,.\, \text{arc. tang.}\, x = \text{arc. tang.}\, x \,.\, \int X\,dx - \int \frac{dx \int X\,dx}{1+x^2},$$

$$\int X\,dx \,.\, \text{arc. sec.}\, x = \text{arc. sec.}\, x \,.\, \int X\,dx - \int \frac{dx \int X\,dx}{x\sqrt{x^2-1}},$$

$$\int X\,dx \,.\, \text{arc. sin. vers.}\, x = \text{arc. sin. vers}\, x \,.\, \int X\,dx - \int \frac{dx \int X\,dx}{\sqrt{2x-x^2}},$$

$$\int \frac{dx}{a + b \cos. x} = \frac{1}{\sqrt{a^2-b^2}} \text{arc. tang.} \frac{\sin. x \,.\, \sqrt{a^2-b^2}}{a \cos. x + b},$$

wenn a — b positiv ist,

$$= \frac{1}{\sqrt{b^2-a^2}} \log. \frac{a \cos. x + b + \sin. x \,.\, \sqrt{b^2-a^2}}{a + b \cos. x},$$

wenn b — a positiv ist,

$$\int \frac{dx \sin. x}{a + b \cos. x} = \frac{1}{b} \log. \frac{a+b}{a + b \cos. x},$$

$$\int \frac{dx \cos. x}{a + b \cos. x} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{a + b \cos. x},$$

$$\int \frac{dx}{(a + b \cos x)^2} = \frac{1}{a^2-b^2} \left[-\frac{b \sin. x}{a + b \cos. x} + a \int \frac{dx}{a + b \cos. x} \right],$$

$$\int \frac{dx \cos x}{(a + b \cos. x)^2} = \frac{1}{a^2-b^2} \left[\frac{a \sin. x}{a + b \cos. x} - b \int \frac{dx}{a + b \cos. x} \right].$$

VIII. Logarithmische und exponentielle Differentialien.

$$\int X\,dx\,.\,\log.X' = \log.X'\,.\int X\,dx - \int\frac{dX'\,.\int X\,dx}{X'},$$

wo X und X' algebraische Funktionen von x sind.

$$\int X\,dx\,.\,\log.x = \log.x\int X\,dx - \int\frac{dx\int X\,dx}{x},$$

$$\int x^m dx\,.\,\log x = \frac{x^{m+1}}{m+1}\left(\log.x - \frac{1}{m+1}\right),$$

$$\int(a+bx)^m dx\,.\,\log.x$$

$$= \frac{(a+bx)^{m+1}}{(m+1)b}\log.x - \frac{1}{(m+1)b}\int\frac{dx\,(a+bx)^{m+1}}{x},$$

$$\int\frac{dx}{x}\log.x = \tfrac{1}{2}\log.^2 x,$$

$$\int\frac{dx}{a+bx}\log.x = \frac{1}{b}\log.x\,.\,\log.(a+bx) - \frac{1}{b}\int\frac{dx}{x}\log.(a+bx),$$

$$\int x^m dx\,\log.(a+bx) = \frac{x^{m+1}}{m+1}\log.(a+bx) - \frac{b}{m+1}\int\frac{x^{m+1}dx}{a+bx},$$

$$\int\frac{dx}{x}\log.(a+bx) = \log.a\,.\,\log.x + hx - \frac{h^2x^2}{2^2} + \frac{h^3x^3}{3^2} - \ldots$$

$$= \tfrac{1}{2}(\log.bx)^2 - \frac{1}{hx} + \frac{1}{2^2h^2x^2} - \frac{1}{3^2h^3x^3} + \ldots$$

wo $h = \frac{b}{a}$ ist,

$$\int x^m dx\,.\,\log^n x$$

$$= \frac{x^{m+1}}{m+1}\Big[\log.^n x - \frac{n}{m+1}\log.^{n-1}x + \frac{n(n-1)}{(m+1)^2}\log.^{n-2}x - \frac{n(n-1)(n-2)}{(m+1)^3}\log.^{n-3}x + \ldots\Big],$$

$$\int\frac{x^m dx}{\log.^n x} = -\frac{x^{m-1}}{(n-1)\log^{n-1}x} - \frac{(m+1)x^{m+1}}{(n-1)(n-2)\log.^{n-2}x} - \frac{(m+1)^2x^{m+1}}{(n-1)(n-2)(n-3)\log.^{n-3}x} - \ldots + \frac{(m+1)^{n-1}}{(n-1)(n-2)\ldots3.2.1}\int\frac{x^m dx}{\log.x},$$

$$\int\frac{dx}{x}\log.^n x = \frac{1}{n+1}\log.^{n+1}x,$$

$$\int x^m dx\,\log.x = \frac{x^{m+1}}{m+1}\left(\log.x - \frac{1}{m+1}\right),$$

$$\int \frac{x^m\,dx}{\log. x} = \int \frac{dy}{\log. y} \text{ für } y = x^{m+1},$$

$$\int x^m\,dx \log.^2 x = \frac{x^{m+1}}{m+1}\left(\log.^2 x - \frac{2}{m+1}\log. x + \frac{2}{(m+1)^2}\right),$$

$$\int \frac{x^m\,dx}{\log.^2 x} = -\frac{x^{m+1}}{\log. x} + \frac{m+1}{1}\int \frac{x^m\,dx}{\log. x}.$$

$$\int a^x \,.\, x^n\,dx = \frac{a^x \,.\, x^n}{\log. a} - \frac{n\,a^x \,.\, x^{n-1}}{\log.^2 a} + \frac{n\,(n-1)\,a^x\,x^{n-2}}{\log.^3 a}$$
$$- \frac{n\,(n-1)\,(n-2)\,a^x\,x^{n-3}}{\log.^4 a} + \ldots \pm \frac{n\,(n-1)\,(n-2)\ldots 3\,.\,2\,.\,1\,.\,a^x}{\log^{n+1} a}$$

$$\int \frac{a^x\,dx}{x^n} = -\frac{a^x}{(n-1)\,x^{n-1}} - \frac{a^x \log. a}{(n-1)\,(n-2)\,x^{n-2}}$$
$$- \frac{a^x \log.^2 a}{(n-1)\,(n-2)\,(n-3)\,x^{n-3}}$$
$$- \ldots - \frac{a^x \log.^{n-2} a}{(n-1)\,(n-2)\ldots 3\,.\,2\,.\,1\,.\,x}$$
$$+ \frac{\log.^{n-1} a}{(n-1)\,(n-2)\ldots 3\,.\,2\,.\,1}\int \frac{a^x\,dx}{x},$$

$$\int a^x\,dx = \frac{a^x}{\log. a},$$

$$\int a^x \,.\, x\,dx = \frac{a^x \,.\, x}{\log a} - \frac{a^x}{\log.^2 a},$$

$$\int a^x \,.\, x^2\,dx = \frac{a^x \,.\, x^2}{\log. a} - \frac{2\,a^x \,.\, x}{\log.^2 a} + \frac{2\,a^x}{\log.^3 a},$$

$$\int a^x \,.\, x^3\,dx = \frac{a^x \,.\, x^3}{\log. a} - \frac{3\,a^x \,.\, x^2}{\log.^2 a} + \frac{6\,a^x \,.\, x}{\log.^3 a} - \frac{6\,a^x}{\log.^4 a},$$

$$\int \frac{a^x\,dx}{x} = \log. x + x \log. a + \frac{(x \log. a)^2}{1\,.\,2^2} + \frac{(x \log. a)^3}{1\,.\,2\,.\,3^2}$$
$$+ \frac{(x \log. a)^4}{1\,.\,2\,.\,3\,.\,4^2} + \ldots$$

$$\int \frac{a^x\,dx}{x^2} = -\frac{a^x}{x} + \log. a \,.\int \frac{a^x\,dx}{x},$$

$$\int \frac{a^x\,dx}{x^3} = -\frac{a^x}{2\,x^2} - \frac{a^x}{2\,x}\log. a + \tfrac{1}{2}\log.^2 a \,.\int \frac{a^x\,dx}{x},$$

$$\int e^{ax}\,dx \sin. x = \frac{e^{ax}}{a^2+1}\,(a \sin. x - \cos. x),$$

wo $\log. \text{nat. } e = 1$,

$$\int e^{ax}\,dx \sin.^2 x = \frac{e^{ax} \sin. x}{a^2+4}\,(a \sin. x - 2 \cos. x) + \frac{2\,e^{ax}}{a\,(a^2+4)},$$

$$\int e^{ax}\,dx \cos. x = \frac{e^{ax}}{a^2+1}\,(a \cos. x + \sin. x),$$

$$\int e^{ax} dx \cos.^2 x = \frac{e^{ax}}{a^2 + 4} \cos. x \ (a \cos. x + 2 \sin. x)$$

$$+ \frac{2 e^{ax}}{a (a^2 + 4)},$$

$$\int e^{ax} dx \sin. bx = \frac{e^{ax}}{a^2 + b^2} (a \sin. bx - b \cos. bx),$$

$$\int e^{ax} dx \cos. bx = \frac{e^{ax}}{a^2 + b^2} (a \cos. bx + b \sin. bx).$$
