VIII.

Map-projections.

Before proceeding further it will be convenient to give a synopsis of the map-projections employed before the end of the 16th century. In doing this, I make use of the word projection in the same extensive sense as that which it has generally obtained. I signify by it not only delineations of maps obtained according to the principles of perspective or by strictly following certain mathematical rules, but generally every method of representing the earth's spherical surface on a plane for geographical purposes, even when the drawing is altogether dependent on conventional rules. It cannot, however, be expected that a detailed description of different kinds of projections or a mathematical development of the theme should be given here. On this subject I would direct the geographer who has no time or opportunity to consult the voluminous original literature on this subject, to: D'AVEZAC, *Coup d'oeil historique sur la projection de cartes de géographie* (*Bullet. de la Société de Géographie*, Paris 1863), A. GERMAIN, *Traité des projections des cartes géographiques*, Paris 1868, H. GRETSCHEL, *Lehrbuch der Karten-Projectionen*, Weimar 1873, and MATTEO FIORINI, *Projezioni delle carte geografiche*, Bologna 1881. The last mentioned work, to which an atlas of the different methods of projection is added, is exhaustive, and is founded on an extensive knowledge of the old literature. I will here only, with constant references to the maps, try to give a chronological review of the projections which, until 1600, have been not only proposed, but actually used. Without such a review it would be difficult duly to understand and appreciate the work of many a distinguished early cartographer. In addition to this, the rich collection of old printed maps at my disposal has given me an opportunity of correcting several errors in modern literature on this subject.

If the period, when a projection was first employed for construction of a geographical map, is taken as a basis for classification, the projections of maps may be divided into the following groups:

A. Projections used before the beginning of the 15th century.

1. *Paratopical* maps. By this name, derived from *παρά* and *τόπος*, I indicate the first incomplete attempts to give a geographical picture of the earth or of some part of it, undertaken without any idea of its globular form, without any certain projection, any graduation or any system of loxodromes. Such were probably all the maps drawn before the time of Hipparchus and Eratosthenes, as well as all maps, based neither on the principles of Ptolemy, nor on sailing directions. Maps of this kind were only exceptionally published in print, as for instance the two maps in the *Rudimentum Novitiorum*, Lübeck 1475, of which the first reproduced by me on fig. 2, is the only paratopical map, which, as far as I know, has been published to serve as a general map of the world; the second (fig. 3), is a map of Palestine, scarcely more successful. A map of this kind is also inserted into the *Sanctarum peregrinationum in montem Syon ... opusculum*, by BERNARDUS DE BREYDENBACH. It embraces (in the edition *per Petrum drach civem Spirensem impressum* MDCCCCXC) Palestine with surrounding lands, from Lebanon to Mecca and Alexandria. The same work also contains other geographical drawings, forming a mean between harbour sketches and views of towns.

2. *Sea-charts or portolanos of the Middle Ages.* Even in these maps the usual geographical coordinates are wanting. Yet the reciprocal positions of the harbour-places have, at least for the main part of the map, been determined with great care by means of the distances and the azimuths of the course-lines in sailing from one place to another. As I have previously (p. 43—51) given a more detailed account of these masterpieces in cartography, it is unneccessary here to return to this subject. The importance of the matter may, however, be an excuse, if I again point out that portolanos, and probably very excellent portolanos, had evidently been already drawn before the use of the compass became general among the navigators in the Mediterranean and Black Seas; that compass-lines had consequently not constituted any original and characteristic feature of these charts, and that the custom of employing loxodrome-lines on maps had probably been introduced independently of the employment of the compass in navigation. It is even possible that these maps had originally been graduated, though the meridians and parallels were gradually exchanged for course-lines or loxodromes, which, at a time when the cosmographic ideas, even of learned men, were very confused and obscure, were not only of far more practical importance to merchants and shipowners than the meridians and parallel lines, but also greatly facilitated the copying of the maps. However, the portolano-draughtsman GRAZIOSO BENINCASA of Ancona, as early as the middle of the 15th century, began to introduce — or to restore (?) — geographical graduation (FIORINI, cited work, p. 353). But even if the portolanos had originally been founded on graduated maps, resembling those of Ptolemy, and constructed with due regard to the globular form of the earth, it is probable that the medieval *constructors* of such maps generally concurred with the most learned ecclesiastics in considering this doctrine as a dangerous heretical error. Nearly all the portolanos still extant are therefore nothing but plane maps (*cartes plates*, D'AVEZAC), only differing from the maps in the preceding group through the rich material at the disposal of the draughtsmen, and, above all, through the care with which this material was arranged and employed by generations of them.

When the navigators of Southern Europe, in the 15th century, had extended their voyages to the other side of the Equator and crossed the Atlantic, there was no longer any possibility, either for map-draughtsmen or navigators, to maintain the old idea of regarding the earth as a flat disk. In spite of this, and although far greater difficulties presented themselves than on the Mediterranean portolanos, in getting distant harbours and towns laid down in their proper places,

the old medieval maps also served as models for the first charts of the whole Atlantic ocean. This could only be effected by sacrificing the correctness of the distances and azimuths, and of the longitudes, whose determination was very difficult before the invention of reliable chronometers. The first printed charts, »*Hydrographiæ sive chartæ Marinæ*,» thus form maps intermediate between true portolanos and maps having a rectangular net of graduation. How obscure the ideas even of very distinguished map-draughtsmen of this period were regarding the mathematical principles of their work, is shown from the circumstance that a common scale was given even for such maps, embracing a large extension in north and south of the earth's surface, e. g. on the Tabulæ novæ in Ptolemaeus 1513, here reproduced on T. XXXV and XXXVI, on the map of the world in Reisch's Margarita of 1515 (N. T. XXXVIII), and on the small map in Medina's *Arte de Navegar* of the middle of the 16th century.

3. *Zone-maps.* These maps form a transition from paratopical maps to maps intersected by a complete net of graduation. Here the meridians are altogether wanting, while the parallels, or at least the lines separating the climates are indicated. The map of the earth, or rather of the old hemisphere, is placed within a circle, inside the circumference of which a space is generally left for the currents of the Ocean. Many an ancient map, now lost, might have belonged to this group, and likewise the majority of medieval maps, which neither directly nor indirectly are based on the works of Ptolemy or of Marinus of Tyre. Generally these maps only form roughly designed drawings of the earth, void of details. They are found in geographical compendiums printed during the 15th century and in uncritical copies of them of a later period. A few such maps are here reproduced on pl. XXXI from the works of Macrobius, Escuidus, and Sacrobosco. To this category also belongs the map in Pierre d'Ailly's *Ymago Mundi* (fig. 19), drawn in about 1410, perhaps after an original by Roger Bacon of the 13th century, and published in about 1483. As stated on the legend at the top of the map, it was not finished, »*quia particularior distinctio majorem figuram requirit.*»

Fig. 20 shows a still ruder drawing, illustrating the relative positions of Europe, Asia, and Africa often met with in medieval manuscripts and in prints of the 15th century.

I know of no zone-map on which any part of the new world was laid down. But the discoveries of the Portuguese receive attention on the zone-maps in *Aristotelis Meteorologia*, Norimbergæ 1512 (N. T. XXXI), and in *La Salade nouvellement imprimée*, Paris 1522. The latter map (fig. 18) is evidently a copy of an original of the 15th century.

4. *Maps on the projection of Marinus* (Ptolemy's equidistant-rectangular, or equidistant-cylindrical projection). For the sake of brevity I shall name this projection after Marinus of Tyre, who, according to Ptolemy, used it for his charts, but I suppose that it had already been employed by earlier unknown cartographers. The meridians and parallels are here equidistant straight lines, forming right angles to each other, and so drawn, that the proper ratio between the degrees of latitude and longitude are maintained on the map's mean or main parallel. When the Equator is selected for this purpose, the net of graduation becomes quadratic. The 26 special maps in all older manuscripts of Ptolemy are drawn on this projection, and likewise the Tabulæ Novæ added to the Latin translation of Jacobus Angelus, for instance the map of Claudius Clavus of 1427 (fig. 27), and the modern maps of the manuscript in Brussels (N. p. 56). When these Tabulæ Novæ were printed, the projection of Marinus was sometimes maintained, even in editions in which Ptolemy's own maps had been reduced to the Donis projection (e. g. *Germania Nova* in edit. 1507, N. fig. 13). The projection of Marinus is also used for most of the maps in the first atlas of the New World, Wytfliet's *Descriptionis Ptolemaicæ Augmentum*, from the end of the 16th century, as well as for maps of minor territories of a much later period. Many charts, embracing extensive parts of the oceans, were drawn on this projection even in the last century, notwithstanding its unfitness for such a purpose, e. g. Waghenaer's charts of the end of the 16th century; *De Lichtende Columne ofte Zee-Spiegel*, published by Jan Jansz. in 1653 and by Pieter Goos in 1658; most of the sea-charts in the famous marine-atlas of van Keulen of the end of the 17th and the beginning of the 18th century; and even many of the charts in Renard-Ottens' atlas of 1745. But in printed editions of Ptolemy this projection was only used for his own maps in Berlinghieris versified Italian translation edited at Florence in about 1478, and for the map of Taprobane in Ptolem. 1478, 1490, 1507, and 1508.

5. *Maps on a conical projection.* Notwithstanding that Ptolemy, in his geography, mentions several different methods of delineating the earth's spherical surface on a plane, he only uses two projections for his maps, viz. the conical projection and the projection of Marinus; the former for the map of the world, the latter for the special maps. In his general map the northern hemisphere is projected on a cone, touching the earth's surface about the parallel of Rhodes, and with a height so calculated, that the proper ratio between the degrees of longitude at the Equator and at Thule (Lat. 63° N.) is maintained. In order to lay down on the same map the »oikumene» of the southern hemisphere, the projection is here modified in a manner explained in chap. XXIV of the first book (comp. p. 5) and which, without further description will be easily understood by a glance at the map (N. T. I). Also on Ruysch's map of the world (N. T. XXXII) the northern hemisphere and the southern one as far as lat. 37° S. are laid down on a cone with the Equator as the base and its fourth part as the lateral height. The vertical height of the cone thus becomes 1,2113 × the radius of the base, and, no regard being given to the polar compression, it intersects the surface of the earth at the Equator and at Lat. 79° 5′. It is accordingly the first instance of an intersecting conical projection. But a glance at the map will suffice to show that the southern hemisphere is thrown considerably out of shape by such an attempt to project larger parts of both hemispheres on the surface of a single cone. This projection is evidently not at all adapted for such maps, and Ruysch's map is the single instance I know of employing an unmodified conical projection for a general map of the world. For special maps, on the contrary, the conical projection has often been used and still continues to be used in mapping more or less extensive parts either of the northern or of the southern hemisphere. This was already done in the edition »Bonnoniæ 1462» of Ptolemy's geography, without, however, the name of the cosmographer being given, who introduced the remarkable modification in the method of projecting the earth's surface on a plane. But it is probable that the passage in the colophon about the share of the »accomplished astrologers» Hieronimus Manfredus and Petrus Bonus, in the redaction of the work, is to be referred to the revision of the maps. For as far as I know the conical projection was never used on the special maps of any of the Greek or older Latin manuscripts,[1] and if I except the map of the world in Ptolemy's geography, for which, as I

[1] Concerning a supposed codex to the Bologna edition, compare p. 12.

have before mentioned, a kind of conical projection was generally adopted, it was long before the example of the editors of the Bologna edition was followed, as may be seen by the following enumeration of the maps printed during the 15th and 16th centuries on a conical projection:

»1462» (1472): The maps in the above mentioned Bologna edition of Ptolemy's geography.

1478, 1490, 1507, 1508: The map of the world in editions of Ptolemy's geography for these years. The same projection was also used for the maps of the world in Pomponius Mela of 1482 (N. T. XXXI), and in the different editions of Schedel's *Liber chronicarum* 1493—1500 (N. p. 38).

1507 and 1508: Ruysch's map of the world (N. T. XXXII).

1558: The above (p. 57) mentioned Zeno map in the work published by Marcolini in Venice in that year (N. fig. 29). We have here a portolano of the northern countries graduated on the conical projection. The net, however, as a conical net of graduation, is not quite correct.

1561, 1562, 1564, 1574: The Zeno map slightly modified, printed from the same plate in the editions of Ptolemy of these years.

1596, 1598, 1621: Reprint of the last mentioned modification of the Zeno map in the editions of Ptolemy of these years.

1564 (?): »*Anglia, Scotia et Hibernia*,» »*Suecia et Norvegia cum confiniis*,» »*Russia cum confiniis*,» by Gerard Mercator, in Rumold Mercator's atlas of 1595. These maps are drawn on a slightly modified conical projection. The cone, on which the map of *Anglia* is developed, intersects the surface of the earth at Lat. 50° and 60°. Corresponding numbers for the map of *Suecia et Norvegia* are 60° and 70°, and for *Russia* 50° and 65°. Mercator's net of graduation is here almost identical with that employed by De l'Isle for the map of Russia of 1745.

According to d'Avezac (*Coup d'oeil* etc., p. 61) the first printed map on an *intersecting conical* projection should be Mercator's map of Europe of 1554, but this is not correct, Ruysch's map having been, as mentioned above, already drawn on that projection. It is even possible that G. Mercator's celebrated map of Europe, which is now lost, was not delineated on the projection supposed by d'Avezac, but on the same projection as: *Europa ad Magnæ Europæ Gerardi Mercatoris P. imitationem edita*. Exact measurements on this map show that it is drawn on a modification of Werner's 2d projection (or a pseudo-conical projection) obtained by making the parallels describe circles with the pole as centre; the meridians straight lines, converging at a point beyond the pole, and preserving at 40° and 60° the proper ratio between the degrees of longitude at these parallels and the degrees of latitude. Such a modification, though less manifest, also occurs on the above enumerated maps of Mercator, who strictly speaking never constructed maps on a true conical projection. Gastaldi, as far as I know, never used conical projections: neither did Ortelius, at least not in his first edition of *Theatrum Orbis terrarum*.

B. Projections of maps introduced during the 15th century.

6. *The projection of Donis* (*Projection trapeziforme:* d'Avezac). This projection, characterized by equidistant, rectilinear parallels and rectilinear meridians converging towards the poles, was first employed for manuscript maps by Dominus Nicolaus Germanus, commonly, though erroneously, called Donis. For want of another name common to different languages, I have distinguished this projection by giving it the name by which its inventor (?) is generally designated. That it actually was »Donis» who first employed this important improvement of the original rectangular projection, seems to be proved by the above (p. 14) extract from his dedication to pope Paul II in the edition printed at Ulm in 1482.[1] Yet this projection had already been used in the Rome edition of 1478 for the special maps (with the exception of the last map of *Taprobane*). As the work of Donis, before being printed, existed in numerous manuscript copies, I conjecture that a manuscript of Donis was even used for the edition of Schweinheim-Buckinck. The projection was much liked, and it has since been used, not only for most maps in the subsequent editions of Ptolemy, but also in several other atlases, e. g. for many of the maps in the *Theatrum Orbis Terrarum* of Ortelius, and in Mercator's *Atlas*, as well as for quite modern maps of small districts. The Ptolemaic maps reproduced in fac-simile by me on pl. II—XXVI; the map of Scandinavia of which I have given a copy on pl. XXX; the Donis map of Scandinavia printed Ulmæ 1482 and 1486; the first map of Scandinavia engraved in copper (N. fig. 14); the map of South America (»Peru») by Forlani of 1566 (N. fig. 80); and the map of Africa by Ortelius of 1570 may be cited as examples of maps drawn on this projection. A special modification of the Donis projection was used on the fine copper-printed maps of Africa IX, X, XI, and XII in Livio Sanuto's *Geographia*, Vinegia 1588, one of which is here reproduced on fig. 50.

7. *The homeother-projection of Ptolemy.* At the end of the first book, Ptolemy further develops the principles of a homeother projection, on which the true proportion between the areas is maintained. According to Ptolemy's rules for the construction of his net of graduation (N. p. 5), the meridians should be drawn so that the parts of the parallel-circles situated between two meridians always preserve the exact ratio to the scale of latitudes. Ptolemy, however, looked upon it as sufficient for practical purposes to maintain this ratio at the equator (e), and at the parallels through Thule (t), Syene (s) and Meroe (m). These quantities are estimated thus:

e: t: s: m: = 5:	$2\frac{1}{4}$	: $4\frac{1}{2}\frac{1}{12}$:	$4\frac{1}{2}\frac{1}{3}$
or 1:	0,45	: 0,902:	0,967
should be 1:	0,454:	0,915:	0,959.

The agreement is here as complete as the system adopted in Ptolemy's geography for designating smaller parts of degrees admits of.

In the old codices this mode of constructing maps seems never to have been employed. But it is used for maps of the world in the editions for which Donis (N. T. XXIX), Münster, Porro, and Keschedt have constructed or engraved plates (comp. p. 7). It was also used for the map in Reisch's Margarita of 1503 (N. T. XXXI), for the maps in several later editions of this work, for the map in the *Cosmographia Pii Papæ*, Parrhisiis 1509 (see p. 40), and also, although in a modified form, for the maps mentioned below under Nos. 8—11.

[1] In the *Coup d'oeil sur la projection des cartes*, Paris 1863, p. 43, d'Avezac mentions a Ptolemy codex (No. 1401 in the *Bibliothèque Nationale* at Paris) »*qui est réputé du XIV:me siècle*,» and in which most maps are drawn on the projection of Donis. If there is no error either in the age or in the projection ascribed to these maps, the statement of Dominus Nicolaus Germanus, that he was the inventor of the new projection, cannot be correct.

C. Projections introduced during the first half of the 16th century.

8. *Stobnicza's homeother projection.* This is but a variety of the preceding one, characterized by a homeother-map of the New World being added to the old homeother map of Ptolemy's Oikumene. The general map of the earth's surface is here for the first time divided into two hemispheres, one of which embraces the Old World from 0° to 180° of longitude east of *Insulæ Fortunatæ*, and the other the newly discovered world between 0° and 180° west of the same departure. The new hemisphere comprises the greater part of the oceans, America and easternmost Asia, *Asia extra Ptolemaeum* or *Asia Marci Pauli Veneti.* The execution of Stobnicza's map is technically very rude, but his projection seems to me to be the best and most accurate of the map projections used during the first part of the 16th century for general maps of the world. With some modifications it might still be used with advantage for general maps on which the earth's surface is divided into two hemispheres. The general outlines of the continents on such a representation are less distorted than on the heart-shaped maps which were formerly so much esteemed; even less than on maps of the world drawn on a stereographic projection, or on the cylindrical projections now generally used. Yet the example was not followed. The only early map constructed in this manner is the map in the edition of 1512 of Stobnicza's *Introductio in Ptholomei Cosmographiam* (N. T. XXXIV), of which I have given a detailed account above (p. 68).

9. *The cordiform projection of Sylvanus.* The only printed map of this class is the general map in the edition of Ptolemy, Venetiis 1511, by Bernardus Sylvanus (N. T. XXXIII). This projection differs from Ptolemy's homeother-projection only in the common centre of the parallel-circles being placed 100° instead of 181° 8′ from the Equator, and in the parallel-circles being so extended, that they embrace not only the space between Long. 0° and 180°, but the whole surface of the earth, with the exception of the segment between Long. 250° and 290°, east of the *Insulæ Fortunatæ*. The map, in consequence of this extension, assumes a curvature at the pole which gives it a cordiform appearance. This would

50. Map of Africa by Livio Sanuto, Vinegia 1588. (Orig. size 397 × 517 m. m.).

[1] Jomard gives a fac-simile of an elegant map engraved on the cover of a metal casket from the 16th century, signed »*Paulus Ageminius faciebat*». This map is a copy of that of Bernardus Sylvanus (compare: Jomard's Atlas, Tab. XVIII; Fiorini, cited work, p. 592).

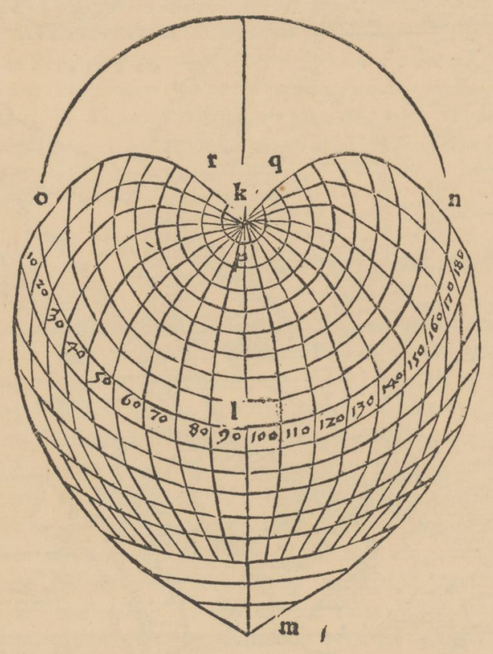
have been still more striking, had not the space between lat. 40° S. and the South pole, i. e. the very point of the heart, been left out.

10. *The cordiform projection of Apianus.* This projection was first used for the map of Apianus of 1520, remarkable as one of the first maps on which the New World is designated with the name of »*America.*» I shall have occasion to return to it in a following chapter. No description of it is given in the works in which it was inserted. But, as far as one can judge from measurements on the map, and on the assumption that the want of continuity in the meridians depends on technical difficulties in the execution, the net of graduation has been constructed according to the principles established by Ptolemy for his homeother-projection. The radius of the equatorial circle is here about 2 × the polar distance (the polar distance being 90° Ptolemy's projection will require 181° 50′). The parallels form concentric equidistant circles, and the proper ratio seems to have been retained between the longitude of the Equator, of the Polar-circle, and of the mean-meridian.

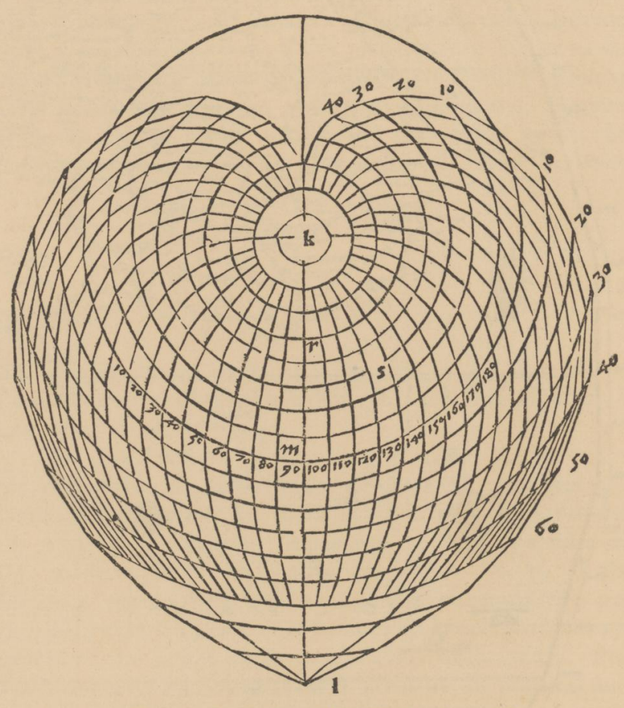
It thus only differs from Ptolemy's projection through Apianus making the map embrace the whole earth, with the exception of the regions in the vicinity of the South pole, through which the map obtains its heart-shaped form. The following maps are constructed on this projection, or on modifications of it, regarding which the reader is referred to the accompanying fac-similes:

The map of the world by Apianus of 1520 (N. T. XXXVIII); the map of Gemma Frisius in the editions 1544, 1545, 1551 etc. of the cosmography of Apianus (N. T. XLIV);[1] the map of the world in Honter's cosmography (N. T. XLIV); Vopel's map of the world in Girava's cosmography, Milano 1555 (N. T. XLV); a gigantic map of the world by Vopel, which I have seen in the Hauslab collection, and which, its size excepted, seems to agree with the map in Girava's work.[2]

11. *Werner's cordiform projections.* Three years after the appearance of the edition of the map of Sylvanus, a work was published in Nuremberg by the celebrated mathematician Johannes Werner, which, besides a critical revision of a part of Ptolemy's geography etc., contained: *Libellus de quatuor terrarum orbis in plano figurationibus ab eodem Joanne Vernero novissime compertis et enarratis.* Werner here[3] gives an account, with necessary numerical tables, of four methods, supposed by him to be new, of projecting the earth's surface on a plane. Three of these are modifications of the homeother-projections mentioned above. They differ from them, because Werner always makes the pole a centre for the parallel-circles, and because he arbitrarily fixes the ratio between the length of the equatorial degree and that of the latitude, while the true proportion between the length of the degrees of the parallels is always maintained.



51. The first projection of Werner, Nurenbergæ 1514. (Orig. size 83 × 110 m. m.).



52. The second projection of Werner, Nurenbergæ 1514. (Orig. size 115 × 132 m. m.).

In Werner's first projection the semi-circle described, with the pole for centre and a radius of 90°, is divided into 180° equatorial degrees. Maintaining the proper proportion between the equatorial- and parallel-degrees we obtain by these means the net of graduation on fig. 51. As may easily be perceived, it is only possible to draw one of the hemispheres on each map constructed on these principles. To avoid such inconvenience Werner, in his second projection (fig. 52), reduces the length of the equatorial degree, assuming it to be the same as that of the degrees of latitude on the mean-meridian.

Werner finally proposes a third cordiform projection, for which, however, no net of graduation has been drawn. This only differs from the preceding one through the length of the equatorial degree being here assumed not to be

[1] The radius of the equatorial circle is here 4 × the distance from the Equator to the Pole. Judging from the rude drawing, the parallel-circles on this map are not equidistant. But as no description accompanies the map it is difficult to decide, whether this is made intentionally or whether it is caused by defective execution.

[2] On Honter's map the radius of the equatorial circle = 1,4, on Vopel's = 3 times the polar distance.

[3] According to Breusing (*Gerhard Kremer gen. Mercator*, Duisburg 1869, p. 45), Johannes Stabius (professor in mathematics at Ingolstadt and Vienna, † 1522) did first set forth the principles of Werner's *second* projection. The works of Stabius have not been at my disposal.

equal to the degrees on the mean-meridian, but $\frac{\pi}{3} \times$ the same.

If l is the distance from the equator to the pole, and q the length of 90° at the equator, the proportion between these quantities will be:

in Werner's first projection $q = \frac{\pi}{2} l$.
in Werner's second projection $q = l$.
and in Werner's third projection $q = \frac{\pi}{3} l$.

Werner himself made no attempt to construct maps on the principles proposed by him. Nor do I know any maps drawn on his first and third projection. But his second projection is strictly applied to the handsome copper-engraving by Joh. Paulus Cimerlinus Veronensis, of which fig. 53 is a fac-simile. It is dated 1566 and inserted into my copy of Lafreri's atlas. The inscription »*Cosmographia universalis ab Orontio olim descripta*» indicates the map to be a copy from a work of Orontius Finæus, the original of which probably was published in 1536[1] at Paris. I am not sure that it is still extant: at least no copies of it are to be found in the British Museum or in the Bibl. Nationale. It is evident that the original map of Finæus also served as a model for the large Turkish cordiform map engraved on wood at Venice by Hhâggy Ahhmed from Tunis, and dated year 967 of the Mohammedan chronology, which corresponds with our year 1559. The blocks for this map which, for some reason or other, had been sequestrated, probably before the issue of the print, were discovered in 1795 in the depositories of the Venetian Council of Ten, and are now preserved at the Biblioteca Marciana. This discovery seems to have produced a certain sensation. It gave rise to a whole literature, introduced by a paper of Abbé Simon Assemani, and to various fables, concerning the manner in which the old blocks came into the possession of the Venetian government. Assemani got permission to draw 24 copies from them, but of these copies very few seem to be now extant. (Comp. D'Avezac, *Bulletin de la Société de Géographie*, Sér. 5: T. 10, 1885, p. 675). To judge from a much reduced copy of the Turkish map given by d'Avezac, it agrees, excepting some unimportant differences, with the map of Cimerlini.

53. Cordiform map of the world by Orontius Finæus. Copper-print by Cimerlinus 1566. (Orig. size 211 × 580 m. m.).

[1] Comp. the biography of Orontius Finæus in Hoefer-Didot, *Nouvelle Biographie Générale.*

Several years before the publication of the cordiform map Orontius Finæus had constructed another map also on Werner's second projection, but modified in such a manner, that the map of the world here is divided into two parts, the one embracing the northern hemisphere with the north-pole for a centre of the parallel-circles, and the other, the southern hemisphere, with the south-pole as a centre. It is of this map that a facsimile is given on pl. XLI. It is dated 1531, but is generally found inserted in *Novus Orbis Regionum ac Insularum veteribus incognitarum*, Parisiis 1532. It was afterwards reprinted from the same block, but with a new title-legend from which the name of Orontius was omitted, in the edition of the Geography of Glareanus printed *Brisgae 1536* and in an edition of Pomponius Mela, *Parisiis apud Christianum Wechelum 1540.*

The map of Orontius Finæus finally had the honour of being copied, although with some modifications, by Gerard Mercator, for one of his first maps, of which I give a facsimile on the plate XLIII from a photo-lithograph of the only known copy, in the library of J. Carson Brevoort (Winsor, *Bibliography of Ptolemy's Geography*, p. 22). Mercator afterwards constructed several special-maps of more or less extensive parts of the earth's surface on the same projection, e. g. his maps of *Africa*, of *Asia ex magna orbis terrae descriptione Gerardi Mercatoris desumpta studio et industria G. M. Junioris*, and probably also of *Taurica Chersonesus* (the southern part of European Russia) in Rumold Mercator's atlas, ed. 1595. This projection is also used for *Asia* in Lorenzo Anania's *Universale Fabrica del Mondo*, 2d ed., Venetia 1582.

Mercator's double cordiform map was reproduced in copper by Anton Lafreri in Rome, probably in about 1560. The fine engraving, of which fig. 54 gives a fac-simile, faithfully follows the original. As to the title-legends the date (1538) is left out from the uppermost legend, and the key to the legends, placed in the original at the edge of *Terra Australis*, replaces the dedication of Gerardus (Mercator) Rupelmundanus to Joannes Drosius on the Roman copy. Here the signature *Ant. Lafreri exc. Romae* is engraved underneath. This reproduction of Mercator's cordiform map is inserted in my copy of Lafreri's atlas, but appears to be rare. It is not to be found in the catalogue of printed maps in the British Museum, nor in Castellani's catalogue of the maps in »*Tavole Moderne di Geografia de la Maggior parte del Mondo*» (Lafreri's atlas) in the library of the Collegio Romano. A copy of this map, belonging to the city-archives of Turin, is described by Fiorini (cit. work p. 622).

12. *The oval projection of Bordone.* The map of the earth is here represented within an oval, of which the shorter axis, formed by the mean-meridian, is of half or about half the length of the longer axis, formed by the equator. The parallels form straight equidistant lines. The meridians are obtained by dividing the equator in parts of the same size, corresponding, for instance, to every 10th degree. The meridians are then drawn from these points of division to the pole in such a way that the parallels also become divided into parts of the same, or almost the same size. The projection is generally, though wrongly, designated by the name of Apianus. Apianus has neither given any description of it nor left any map drawn according to it. In his cosmography we only find a few small wood-cuts with a net of graduation similar to that which belongs to the projection in question. But the figures are not accompanied by any explanation; the meridians are wanting, and the dimensions of the oval are far from correct.

In carefully examining the different oval maps of the world, printed in the 16th century, it will be found that, although apparently much resembling each other, they present notable differences, as well in the form of the oval as in the way the meridians are drawn. On Bordone's map the oval appears to be formed by the line obtained, when an elastic ring is compressed between two parallel planes, until the longer axis becomes twice the length of the shorter one. In the Ptolemæus of Gastaldi, and on the general map of Porcacchi, the oval is formed by two parallel lines and two circle-segments. On other maps arbitrary devices seem to have been applied, in drawing the meridians.

In the 16th century this projection was much used, e. g. on:

A map in *Libro di Benedetto Bordone, nel qual si ragiona del tutte l'Isole del Mondo* etc., Vinegia 1528. The map is supposed to have been finished before 1521 and was reproduced from the same block in several later editions of Bordone's *Isolario*. The Polar-axis (p): Equatorial axis (e) = 1 : 2 (N. T. XXXIX).

A map in *De Orbis Situ Epistola* by Franciscus Monachus, Antverpiæ 1524. p : e = 1 : 2 (According to the much reduced reproduction by Lelewel).

A map in *Novus Orbis* by Grynæus-Huttich, Basileæ 1532. p : e = 1 : 1,87. (N. T. XLII).

A map in the 2d ed. of *Isolario di* Bartolomeo dalli Sonetti, Venetiis 1532 (There is no general-map in the editio princeps). p : e = 1 : 1,87.

A map in Münster's ed. of Ptolemy, Basileæ 1540. p : e = 1 : 2. (N. T. XLIV). Reproduced in several later editions, as well of Münster's Ptolemy as of his cosmography.

Seb. Cabot's large map of the world of 1544. p : e = 1 : 1,333 (according to the fac-simile published by Jomard).

»*Universale Novo*» in Gastaldi's Ptolemy, Venetiis 1548. p : e = 1 : 1,77. (N. T. XLV).

A large map of the world by Gastaldi, of which two editions are described by Fiorini (cit. work, p. 601). On one of them is written: *Universale descriptione di tutta la terra conosciuta fin qui. In Venezia al segno del Pozzo*, and in the dedication of the copper-engraver, Paulo Forlani, dated 1562, *Giaccomo Gastaldo cosmografo raro* is mentioned as the author of the map. The other edition bears the inscription: *Paulus de Forlanis Veronensis opus hoc Cosmographi Iacobi Gastaldi pedemontani instauravit et dicavit Paulo Michaeli Vicentino. Venetiis Ioan. Francisci Camotii aereis formis. Ad signum Pyramidis. Anno MDLXII.* A third edition, not dated, is cited by d'Avezac (*Coup d'oeil* etc. p. 72). These maps, like all maps printed on separate sheets, are now extremely scarce, though once much appreciated and often reproduced.

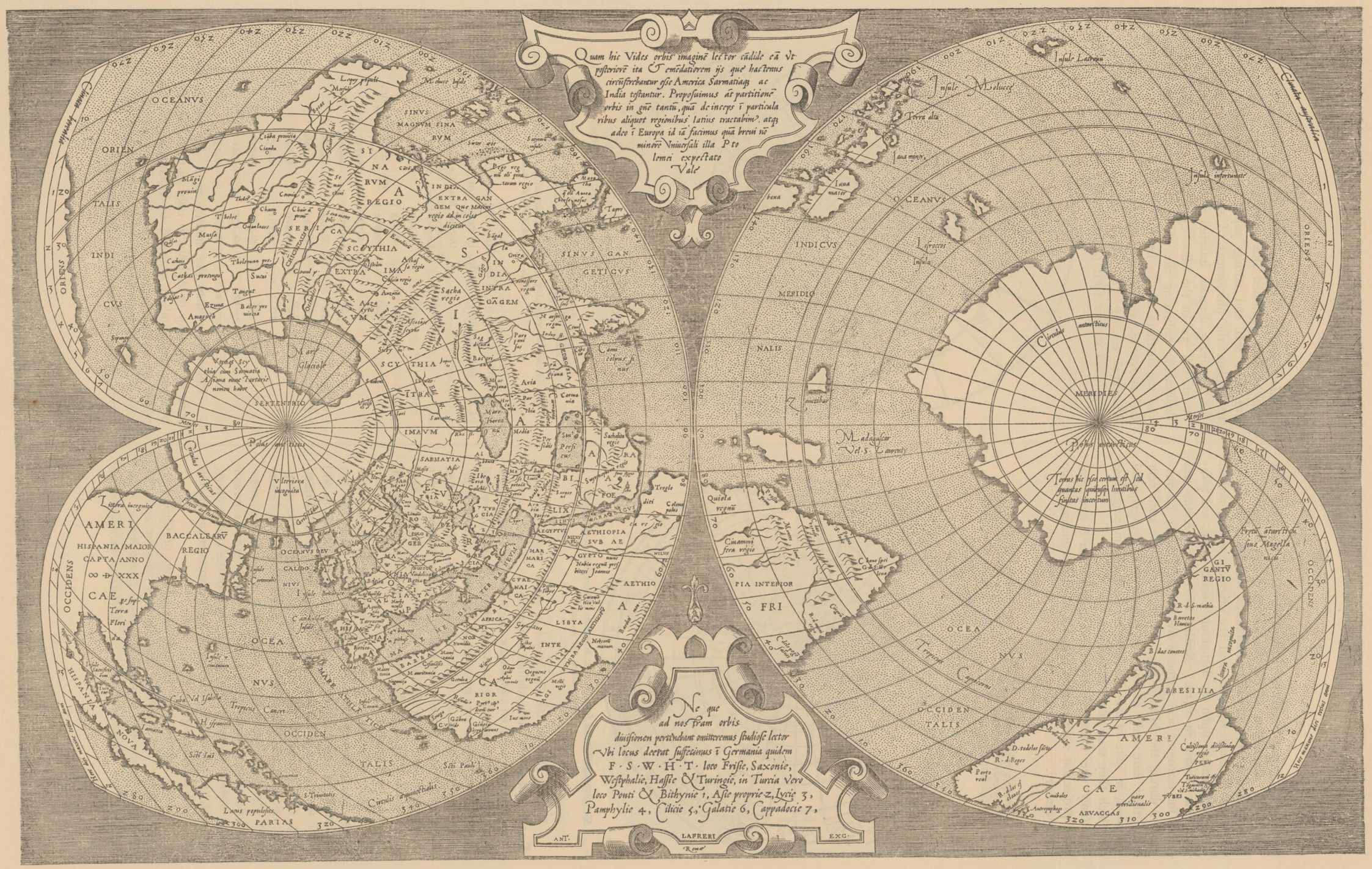
The map of the world in the *Theatrum Orbis terrarum* of Ortelius, Antverpiæ 1570. p : e = 1 : 2. (N. T. XLVI).

The map of the world in the *Opusculum geographicum* of Myritius, Ingolstadii 1590. p : e = 1 : 1,8. (N. T. XLIX).

A map by Hieronymus Porro, in the Ptolemæus Venetiis 1596 of Maginus. p : e = 1 : 2.

Maps of the world on this projection have further been constructed by Battista Agnese (in his portolanos), by Belle-Foreste (*La Cosmographie Universelle*, Paris 1575), by Gioseppe Rosaccio (*Il Mondo e sue parti*, Fiorenza 1595), and others.

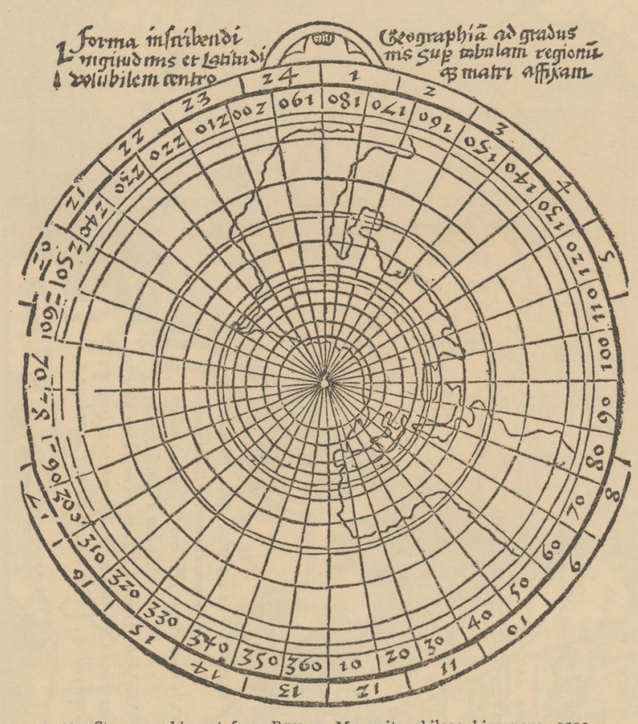
13. *Stereographic projections.* The strictly perspective projections, the centrographic, the stereographic, the scenographic, and the orthographic, are considered to have been invented by Thales, Eratosthenes, and Hipparchus. The most important among them, the stereographic and the orthographic, were under the name of *Planispherium* and *Analemma* described by Ptolemy in two treatises, of which that on the planisphere was first printed in Latin as an appendix to the Rome edition of 1507 of the geography (Comp. N.



54. G. Mercator's double cordiform map of the world of 1538. Copperprint, Rome about 1560. (Orig. size 324 × 519 m. m.).

p. 16). It was a translation from a Greek manuscript. But Ptolemy's description of the Analemma is only known from a very incomplete and faulty translation into Latin (probably from an Arabian translation of the original), which, revised and commented upon by Federigo Commandino, was published at Rome in 1552. Neither of these projections was used for cartographical purposes by Ptolemy, or by his successors during the Middle Ages. Even during the 16th century they were only occasionally employed for geographical purposes, in spite of the elaborate mathematical investigations of which they were often the object.

Only the following maps and map-nets on a stereographic projection were printed previous to the beginning of the 17th century.

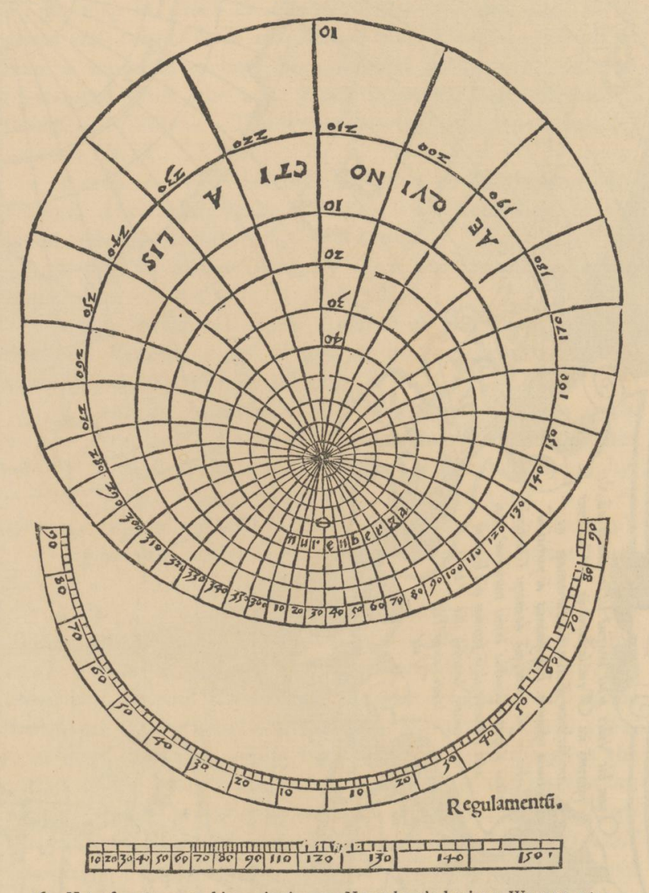


55. Stereographic net from Reisch, Margarita philosophica nova, 1512. (Orig. diam. 131 m. m.).

Reisch, *Margarita Philosophica nova*, Argentine 1512. To this and several later editions of the famous Reisch encyclopedia was appended an *Appendix Matheseos in Margaritam Philosophicam* containing, among tracts on the Greek and Hebrew languages, on architecture and music etc. a *Tractatus de compositione astrolabii Messehalath.* We here find directions as to a polar-stereography, with two drawings of a stereographic net of graduation. On one of them, given here in fac-simile (fig. 55), the commencement of a geographical map can be discerned.

A net of graduation on a stereographic horizontal projection by Johannes Werner 1514. (N. fig. 56.) The last projection of this eminent mathematician, in his above (p. 88) cited work, is such a projection on the horizon of Nuremberg. He praises several of its points, and invites the adoption of it in the following words: *Talis profecto terrarum orbis figuratio, plurimum honestatis atque ingens ornamentum viro adiiciet philosopho, si super ipsius mensæ plano depicta fuerit.* During the whole of the 16th century, however, no attempts were made to construct maps on this variety of the stereographic projection.

A map inserted into the different editions of *Cosmographicus Liber Petri Apiani Mathematici studiose collectus.* (Colophon in the first edition:) *Excusum Landshutæ . . . impensis Petri Apiani Anno . . . Millesimo quingentesimo vicesimo quarto.* This edition of 1524, as well as the numerous editions published afterwards, contain some diagrams cut out from stiff and strong paper, and invented by Apianus to explain the phenomena of astronomical geography. On the revolving diagram inserted at fol. 63 of the edit. 1524, or at fol. 32 of the edits. 1533 and 1534, a small map is delineated, embracing the whole of the northern hemisphere and the southern one to lat. 25° S. (N. fig. 57.) If we except the above mentioned very incomplete drawing in Reisch's Margarita (N. fig. 55), which can hardly be regarded as a map, this is the first printed map on a stereographic



56. Net of a stereographic projection on Nuremberg's horizon, Werner 1514. (Orig. size 124 × 175 m. m.).

polar-projection. The table below shows that the drawing of the net, although rude, is very correct.

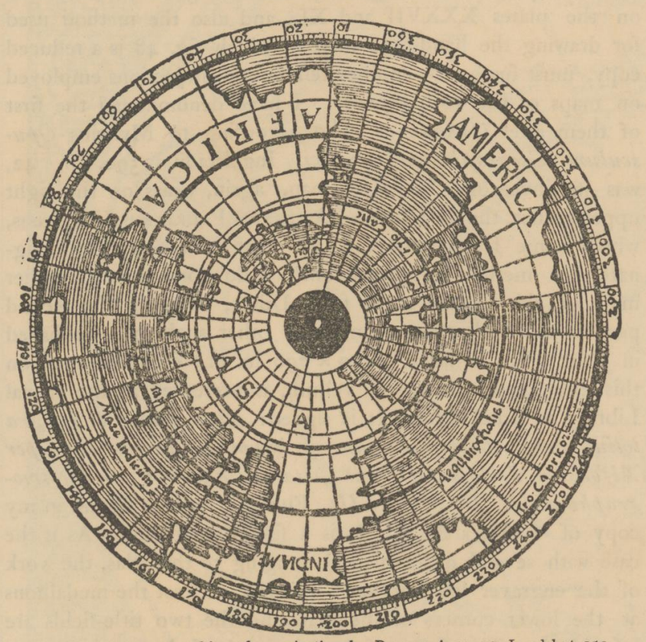
Latitude.	Distance from the Equator. Stereographic polar-projection.	Map of Apianus (edit. 1533).
90°	1,000	1,000
80°	0,913	not determinable.
70°	0,824	
60°	0,732	0,742
50°	0,636	0,632
40°	0,534	0,532
30°	0,423	0,426
20°	0,300	0,292
10°	0,161	0,157
0°	0,000	0,000
—10°	0,192	0,210
—20°	0,428	0,458

Considering that the distance from the equator to the pole on the map of Apianus is only 34 m. m., the agreement between the numbers obtained by calcutation and those obtained by measurement on the map is very satisfactory, and proves that we here actually have before us a map correctly drawn on the stereographic polar-projection.

A stereographic net constructed by Orontius Finæus. This celebrated mathematician described and delineated in his cosmography, published in *Orontii Finaei Delphinatis opus varium*, Parisiis 1532, a net of graduation on a stereographic meridian projection, and a few years later he, in his *Planisphærium geographicum*, Lutetiæ 1544, discussed the advantages of employing a stereographic polar projection for geographical maps. (Fiorini, cit. work, p. 127).

Rumoldus and Michael Mercator. In the first edition of Gerard Mercator's work *Atlantis Geographia Nova Totius Mundi. Authore Gerardo Mercatore Rupelmundano Illustriss. Ducis Iuliæ etc. Cosmographo*, Duysburgi Clivorum s. a. (the preface, dedicated by Rumoldus Mercator to Queen Elizabeth of England, is dated Duisburg first Apr. 1595), there are two maps on the stereographic prejection, viz.

The map of the world by Rumoldus Mercator (N. T. XLVII). In the text printed beneath the map is written: *Sciet lector nos eam conplanandæ sphærae rationem secutos esse,*



57. Map on stereographic polar-projection by Petrus Apianus, Landshut 1524. (Diam. of the orig. 112 m. m.).

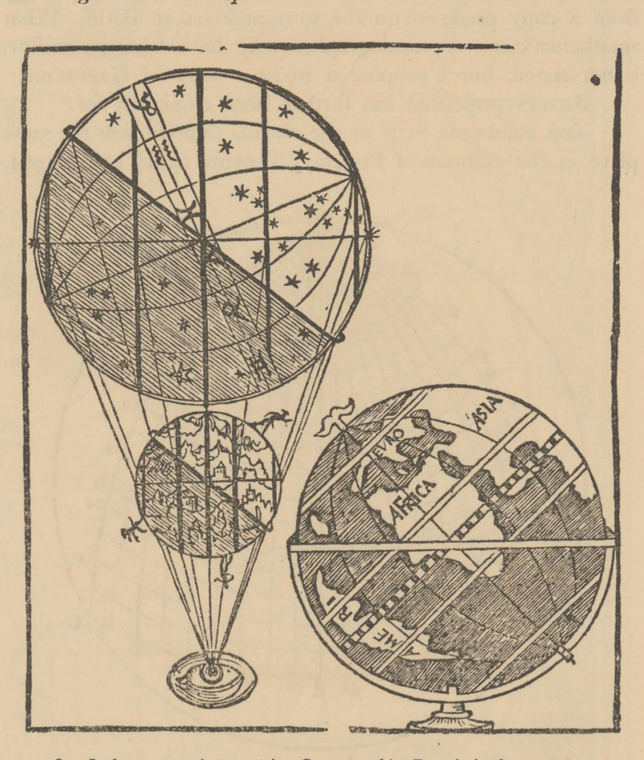
quam Gema Frisius in suo planisphærio adinvenit, quæ omnium longe optima est.[1]

The map *America sive India nova ad magnae Gerardi Mercatoris avi universalis imitationem in compendium redacta per Michaelem Mercatorem Duysburgensem* from the same atlas. Fiorini pretends (cit. work, p. 129) that the map of Africa is also drawn on a stereographic meridian projection. But this is not correct, at least not as regards the handsome map of Africa inserted under Litt. C in my copy of edit. 1595, and constructed on Werner's 2d projection.

Rumold Mercator's map of the world was copied by Hieronimus Porro and published in the Ptolemy of Maginus of 1596, 1598, and 1621, and by Petrus Keschedt in the editions of 1597, 1608, and 1617.

No maps on the orthographic projection were, as far as I know, published before the 17th century, nor any on the scenographic, unless such woodcuts of globes as I have reproduced on pl. XLIV from the cosmography of Apianus, and in fig. 49 from a work of Schöner, are regarded as scenographic representations of the earth.

14. *Bacon's meridian projection.* This projection is described by Roger Bacon († 1294) in the following words: *Sed in signatione civitatis in loco suo per longitudinem et latitudinem suam inventas ab auctoribus, superaddam artificium, quo locus civitatis habeatur per distantiam ejus a meridie et septentrione et oriente et occidente. Et hoc artificium consistit in concursu lineæ rectæ aequidistantis aequinoctiali signatæ in plano: secundum formam lineæ rectæ ductæ a numero graduum latitudinis regionis signato in quarta coluri ducta ab aequinoctiali ad polum mundi in concursum, inquam, cum arcu circuli magni qui transit per polos mundi et per numerum longitudinis civitatis signatum in aequinoctiali circulo. Hic autem modus*



58. »In hoc sequenti typo totius Cosmographiæ Descriptio demonstratur». Petrus Apianus, Landshut 1524. (Orig. size 109 × 126 m. m.).

melior est et facilior, et sufficit considerationi locorum mundi in hujusmodi figuratione sensibili (*Opus Majus*, Ed. 1733, p. 186). In the beginning of the 15th century this remarkable passage was transcribed almost word for word by Pierre d'Ailly in the 17th chap. of his *Compendium cosmographicum*, which has caused d'Ailly to be quite undeservedly cited by Lelewel and Fiorini as the inventor of this projection. (Comp. Lelewel, II, p. 75; Fiorini, p. 604.) The projection is characterized by equidistant, rectilinear parallels and meridians formed by arcs of circles passing the poles and dividing the equator in equidistant parts. The maps of the whole surface of the earth on this projection were generally, when it became practically employed, divided into two hemispheres, and the great interest attached to it from a cartographical point of view, perhaps depends upon such a division of the general map of the earth being thus definitely introduced into cartography. But it should be remembered that in this respect these maps had a precedent in Stobnicza's

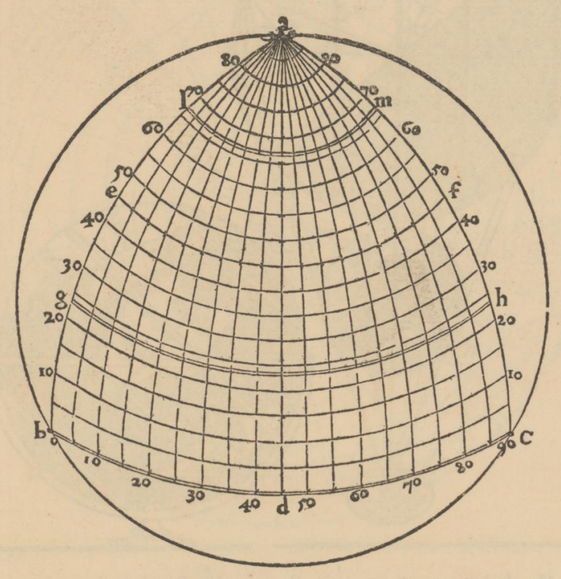
[1] It is evidently a mistake, when this map is ascribed to Rumold's celebrated father Gerard. The title expressly says: *Ex magna Universali Gerardi Mercatoris . . . Rumoldus Mercator fieri curabat.* But here by »Magna Universalis» is probably meant G. Mercator's chart on a cylindrical projection, as would appear from the enumeration of his cartographical works in the introduction to »Atlas» by Gualterus Ghymmius. At present no map drawn on the stereographic projection by G. Mercator sen. is known, but evidently it was he who superintended and inspired the works of his sons and grand-sons.

map of 1512 (N. T. XXXIV), which however seems to have been entirely overlooked by the cosmographers up to our time.

The first[1] map (N. fig. 58) printed on this projection is inserted in the text of the cosmography of Apianus. It is an insignificant woodcut, which was subsequently reproduced on the title-page of *Dionysius Lybicus Poeta De situ habitabilis orbis a Simone Lemnio poeta Laureato nuper latinus factus*, Venetiis 1543. Bacon's projection was further employed on a map of the world, engraved in copper by Giulio Musi and published in Venetia by Tramezini in 1554. On the upper borders of the map we read: *Cum priv. summi pont. et senat. veneti Michaelis Tramezini formis MDLIIII*, and beneath: *Julius de Musis Venet. in aes incidit MDLIII.* I have not seen this map. Fiorini describes it (cit. work, p. 605) from a copy preserved in the town-archives at Turin. What »mathematician» or cosmographer may have been its author, is not stated, but I suppose it to be a work of Gastaldi.

Bacon's projection has further been employed for:

The handsome map of the world, printed from the same plate in the editions of Ptolemy, Venetiis 1561, 1562, 1564,



59. Net of graduation by Orontius Finaeus in 1551. (Orig. size).

and 1574, and in Lorenzo d'Anania's *L'Universale Fabrica del Mondo*, 2d edit.,[2] Venetiis 1582 (N. T. XLV).

Maps of Africa, Asia, Europe, and America in André Thevet: *La Cosmographie universelle,* Paris 1575.

The map No. 2, *Americæ sive Novi Orbis nova descriptio*, in edit. 1570 of the *Theatrum Orbis Terrarum* of Ortelius.

The maps of the two hemispheres at the upper part of the first map in de Judaeis' Atlas, Antverpiæ 1593.

15. *Da Vinci's projection.* The above mentioned geographical drawing, found among the papers of Leonardo da Vinci and described by R. H. Major, is drawn on a peculiar and well characterized projection, as may be seen by the fac-simile of Major's copy given above (fig. 45). I do not know of any other instance of such a construction having been employed for early maps, but a net of parallels and meridians is given in the *Sphæra Mundi* by Orontius Finæus, Lutetiæ Parisiorum 1551 (N. fig. 59).

16 and **17.** *The projections of Glareanus and Florianus.* The method of representing the spherical surface of the earth, employed in the globe-prints here given in fac-simile, on the plates XXXVII and XL, and also the method used for drawing the handsome map of which fig. 48 is a reduced copy, must further be enumerated among projections employed on maps of the 16th century. I have denominated the first of them after Glareanus, who according to Myritius *Opusculum Geographicum Rarum*, Ingolstadii 1590, p. 42, was its inventor. The last map, again, bears on the right upper corner, the medallion of Antonius Florianus Utinensis, which name I have adopted, for want of a better, to designate the method of projection.[3] I do not know any other map of this Italian artist from Udine, living in the second part of the 16th century, nor any other such map published in print. But a large (1440 × 790 m. m.) coloured map on this projection, drawn on vellum, is preserved at the Royal Library in Stockholm. It is signed: *Nova verior et integra totius orbis descriptio nunc primum in lucem edita per Alfonsum de Sancta Cruz*[4] *Cæsaris Charoli V. archicosmographum. A. D. MDXLII.* Floriani's map is bound in my copy of »Lafreri's Atlas.» It is a fine copper-print. As is the case with several other maps belonging to this atlas, the work of the engraver has not been finished, at least the medallions at the lower corners of the map and the two title-fields are left blank. The form of America and of the northern part of the Pacific indicates that the map had probably been engraved before 1566, the year when »*Fretum Anian*» was introduced into cartography.

D. Projections first employed between 1550 and 1600.

18. *Mercator-Postel's equidistant polar projection.* Although possessing many excellent qualities, the stereographic polar-projection suffers from the defect of having the degrees of latitude near the equator considerably larger than those in the vicinity of the pole. On the orthographic polar-projection, on the contrary, the degrees of latitude near the equator are too small. A construction, intermediate between these is that with rectilinear meridians converging towards the pole and circular equidistant parallels. This projection was generally supposed (for instance by d'Avezac, cit. work p. 63) to have been first used by Postel in 1581. But Breusing, (*Gerhard Kræmer gen. Mercator,* Duisburg 1869, p. 51) claims priority for Gerard Mercator, who as early as 1569 employed it for the map of the polar regions, added to his celebrated chart on an isogonic cylindrical projection. I have, therefore, in the Swedish edition of this work desig-

[1] From the above quoted passage of Roger Bacon it may be concluded that he himself constructed a map on the newly-invented projection. This map is lost, but I suspect that the map-skeleton of d'Ailly on this projection (N. fig. 19) is copied from a manuscript of Bacon.

[2] The first edition of this work does not contain any maps, but the 2d edition contains, besides the map of the world, maps of Europe, Africa, and America on the Donis' projection and of Asia on Werner's 2d projection.

[3] This map is mentioned in Castellani's catalogue of the library of Collegio Romano (p. 239) by the following words: *Mappamondo in due emisferi formati a spicchi concentrici ai circoli e ai poli. Negli angoli, su in alto a sinistra è il ritratto di Tolomeo, alla diritta quello del Cosmografo »Antonius Florianus Vtin.»* Castellani may at least have known some biographical data regarding Florianus, as he is called a cosmographer.

[4] The University library of Upsala possesses a large map of the city of Mexico by the same imperial *archicosmographer* Alonzo de Santa Cruz. It is not known how and when these two maps came to Sweden, but probably they belonged to the large collection of maps left by Santa Cruz at his death 1572, and of which a list was deposited in Archivo de Indias at Seville. (*Relaciones geográficas de Indias publicalas el Ministerio de fomento. Perú.* T. 2, Madrid 1885, p. XXX).

nated it by the name of Mercator-Postel, a name which I shall also use here to avoid confusion. But exact measurements, which I have since had occasion to undertake on the special maps in the edition of Ptolemy of 1462, show that most of these maps are drawn on the equidistant polar-projection, which is of course only a variety of the conical development. The name which would most nearly express its origin would, therefore, perhaps be *the projection of Manfredus Bonus.*

the large sea-chart, is in more than one respect of interest in the history of geography.[1] A fine copy of it had already been published in 1593 by De Judaeis in *Speculum Orbis Terræ* (N. T. XLVIII). This projection was also used for the polar-map of Michael Lok, *civis Londinensis*, inserted into Hakluyt, *Divers Voyages*, London 1582, and for the celebrated map of Willem Barents' last voyage, published in Linschoten's *Navigatio ac Itinerarium*, Hagæ-Comitis 1599.

60. G. Mercator's map of the North-polar regions of 1569, from Rum. Mercator's atlas of 1595. (Orig. size 357 × 393 m. m.).

The polar-map of Mercator at first evidently created far more interest than the main chart, with which it was published. It was reproduced by Mercator himself in a copper-engraving inserted into the first edition (1595) of *Atlas*. The metallotype fig. 60 gives a fac-simile of this map, which, although by no means comparable, in importance, with

19. *Mercator's isogonic cylindrical projection (Mercator's projection).* The first map on this projection, which has exercised such powerful influence on the progress of navigation, was published in 1569 by Gerard Mercator. A long inscription on the map explains the principle of the new method of projection and its use for navigation. Mercator is

[1] To what I have said before regarding Mercator's remarkable delineation of the North, I may here add the following extract from the long legend attached to the polar-map on his chart: . . . *Quod ad descriptionem attinet, eam nos accepimus ex itinerario Iacobi Cnoyen Buscoducensis, qui quidem ex rebus gestis Arturi Britanni citat, majorem autem partem et potiora a sacerdote quodam apud regem Norvegiae anno D. 1364 didicit* (Comp. Lelewel, II: p. 231; Jomard's fac-simile). From a passage in Purchas (*His pilgrimes*, III, London 1625, p. 518) we know that Iodocus Hondius possessed a copy of Ivar Baardsons description of Greenland of the middle of the 14th century. This copy probably once belonged to Gerard Mercator, whose copper-plates (and other geographical documents?) Hondius purchased in 1604. From this we may conclude that Ivar Baardson was the Norwegian priest of whom Mercator speaks in the above cited legend — and perhaps we here too have a clue to the origin of the Zamoiski-map (N. T. XXX).

thus incontestably its real inventor, notwithstanding that maps on such a projection, i. e. with rectilinear equidistant meridians and parallels so drawn that a proper ratio is always maintained between the longitudes and the latitudes, had, as I have already (p. 22) pointed out, been promised by Bilibaldus Pirckheimerus in the introduction to Ptolemy, printed at Strassburg in 1525, and although the mathematical principles on which it is based, and the tables necessary for its construction, were first published by Edw. Wright in his important work: *The correction of certain Errors in Navigation detected and corrected*, London 1599;[1] 2d edition 1610.

Of the large map of 1569 there is at present only one copy known, preserved in the Bibliothèque Nationale at Paris. A full-size fac-simile of it was published by Jomard, and a copy on a considerably reduced scale by Lelewel. The map scarcely appears to have been duly appreciated even by Mercator's nearest friends and admirers. It is not, like several other large maps of Mercator, reproduced on a reduced scale in the Atlas. Mercator's friend and biographer, Walter Ghymm, enumerates it among his works, but evidently without any idea of its real importance. Neither Waghenaer, nor Willem Barents employ it for the charts they published during the latter part of the 16th century. The length of time the reform introduced by *Magna Mercatoris* and Wright's *Errors of navigation*, needed for its general adoption is made evident from the circumstance, that all charts in *De Lichtende Columne ofte Zee-Spiegel*, published in Amsterdam by Jan Jansz in 1653 and by Pieter Goos in 1658, are still drawn on the rectangular projection of Marinus. This is also the case with most[2] charts in van Keulen's large atlas and even in the *Atlas van Zeevaert en Koophandel* by Renard-Ottens, Amsterdam 1745. That neither of these celebrated cartographers did fully appreciate the mathematical principles explained by Wright proceeds from the circumstance that the charts in their works, although drawn on the projection of Marinus, and often extending from 50° to 80° of latitude, are yet crossed by compass-bearings, in all directions.

The only printed maps of the 16th century known to me, which are drawn on Mercator's projection are:

1569: Mercator's large map: *Nova et aucta orbis terræ descriptio ad usum navigantium emendata, accomodata. . . . Aeditum autem est opus hoc Duysburgi an. D. 1569 mense Augusto.* Its dimensions (2,0 × 1,26 m.) prevent its reproduction here. A fullsize fac-simile is published by Jomard, but unfortunately with omission of several of the important inscriptions, for which Lelewel's *Géographie du Moyen âge*, II, p. 225, may be consulted.

1599: A map of Henricus Hondius in *Navigatio ac Itinerarium Johannis Hugonis Linscotani* . . . Hagae-Comitis 1599 (N. fig. 61). Among the other maps in this work one *(Delineatio chartæ trium navigationum per Batavos ad Septentrionalem plagam)* is constructed on the equidistant polar-projection.

1599: The handsome map in Richard Hakluyt's *Principal Navigations*, 2d edition (N. T. L), which is supposed to be »the new map» of which Shakespeare speaks in »Twelfth Night» (Act. III, Sc. 2). Mr C. H. Coote suggests that Edward Wright is the true author of this map. It is one of the best general maps of the world of the 16th century (Comp. *The voyages and works of John Davis the Navigator.* By Albert Hastings Markham. Works issued by the Hakluyt Society, London 1880, p. LXXXV).

The following table shows how nearly the constants of the nets of graduations in the oldest maps constructed on Mercator's projection, are calculated.

Distance to the equator in equatorial degrees:

Parallel at	Calculated[3] for Mercator's projection.	On Mercator's map of 1569 (Jomard's copy).	On Hakluyt's map of 1599.	On Hondius' map of 1599.
10°	10,05	10,1	10,1	10,1
20°	20,42	20,3	20,9	20,9
30°	31,47	31,0	31,3	32,0
40°	43,71	42,8	43,1	—
50°	57,91	56,5	57,2	—
60°	75,45	73,3	74,4	—
70°	99,43	96,3	99,0	—
80°	139,59	135,2	139,1	—

As may be perceived, but little remains to desire regarding the agreement between the numbers of the 2d column and the corresponding numbers on Hakluyt's map. On this map the equatorial degree is = 0,55 m. m., and the greatest difference between the calculated and the observed equatorial-distance only 1,05 × 0,55 = 0,6 m. m. At 80° the error amounts to 0,27 m. m. and, at 70°, to 0,24 m. m. On Mercator's map the differences at 10° and 20° are insignificant. At 30° the distance from the equator falls short of the calculated number by 0,47, at 40° by 0,91, at 50° by 1,41, at 60° by 2,15, at 70° by 3,15 and at 80° by 4,39 equatorial degrees. Such a degree has here a length of 1,73 m. m. An error occurs, gradually increasing towards the pole, and evidently arising from the imperfection of the mathematical resources of the map-constructors in the middle of the 16th century. Mercator seems to have calculated the length of the intervals between every tenth degree of the parallel by means of the approximate formula:

$$P_{\varphi+10} - P_{\varphi} = \frac{10}{\mathrm{Cos}\,(\varphi + 5)}$$

The unity here is the length of the equatorial degree, and P_{φ} the equatorial distance on the map at the latitude φ.

By this formula the following numbers are obtained:

	Distance from equator in equatorial degrees.	
The parallel at	Calculated	Mercator's map (Jomard's copy).
10°	10,04	10,1
20°	20,39	20,3
30°	31,42	31,0
40°	43,63	42,8
50°	57,77	56,5
60°	75,20	73,3
70°	98,86	96,3
80°	137,50	135,59

Even here the agreement is not so complete as might have been expected, but the differences can be explained by engraving-errors or by stretchings in the paper.

On printed maps which I have had an opportunity of examing, there have been employed down to A. D. 1600 nineteen different projections. This number might be further increased, if separate numbers were to be given to the pro-

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[1] In his biography of Mercator Ghymmius says with regard to the new projection: . . . *inventio nova et convenientissima . . . quæ sic quadraturæ circuli respondet ut nihil deesse videatur, praeterquam quod demonstratione careat, ut ex illius* (Mercatoris) *ore aliquoties audivi.* It is not clear from this passage whether »*quod demonstratione careat*» concerns the projection or its connection with quadratura circuli. The latter appears to me more probable as fully harmonizing with the tendencies to speculations in the most heterogeneous branches of knowledge, which characterized the great cartographer.

[2] In the edition of 1683 of van Keulen's atlas only the first map (the map of the world) and the last one, of the north-eastern coasts of Asia, from Novaya Zemlya to Japan, are drawn on Mercator's projection. In the atlas of Renard-Ottens only the map of the world is drawn on Mercator's projection.

[3] Assuming the surface of the earth to be spherical.

61. Chart on Mercator's projection in: *Navigatio ac Itinerarium Iohannis Hugonis Linscotani.* Hagae-Comitis 1599. (Orig. size 396 × 490 m. m.).

jections, apparently identical, but yet from a mathematical point of view very different, which have here been associated under No. 12 (Bordone's oval projection), and also to Ptolemy's tangent, and Ruysch's intersecting conical projections, and to the remarkable pseudo-conical or pseudo-Werner projection adopted by Mercator for his map of Europe. On the other hand it would, perhaps, be most correct, with d'Avezac, to unite the homeoter projection of Ptolemy with that of Stobnicza, and to enter the cordiform projections (Nos. 9, 10, and 11) under a common number. This uncertainty arises from the difficulties connected with a strict limitation of the different kinds of early maps. It is an exception to find a mathematical description given of the net of graduation which has been employed, and the maps, especially when reproduced in wood-cut, are often of such inferior execution, that it is difficult to decide, even by careful measurements, what may have been the leading principles for the drawing. I hope, however, that the enumeration given above and illustrated by numerous facsimiles, will be sufficient for a general review of the early history of map-projections.

We learn from it that this chapter of mathematical geography had, at the end of the 16th century, already reached a very high development. It is true that no clear insight into the properties, advantages, and defects of the different projections had yet been obtained, nor was such knowledge then possible, owing to the deficiencies in mathematical resources. So that there could not be a critical inquiry into the conditions which it was possible to satisfy, in representing the spherical surface of the earth on a plane. But several of the most valuable methods of projection, for instance that of the development on an intersecting cone, the stereographic, the isogone-cylindrical, and the equidistant polar-projections, were then in use. Nor can it be said that any radical reform in cartography has been introduced owing to the complete mathematical analysis of the problem, on which the works of the modern cartographers can be based.

The merit of so early a development of the doctrine of projections must, in the first place, be ascribed to Ptolemy. It is true that he was unable to solve those analytical problems on which an exact theory of map-projections must be based. They were then insoluble, and are very difficult even in our days. Nevertheless he clearly understood the truth that the surface of a sphere cannot be exactly developed on a plane, and that consequently the problem must be solved by an approximation, for which he proposed not less than four different methods, two of them being practically applied by himself. At least two of these had already been employed or proposed by his predecessors, Hipparchus and Marinus. This important chapter of mathematical geography was further developed by Arabian writers, by Bacon and Nicolaus Germanus, by the authors of the maps in the Ptolemy edition Bononiæ 1462, by Ruysch, Bernardus Sylvanus, Bordone, Johannes Werner, Petrus Apianus, Glareanus, Orontius Finæus, Postel, and Mercator.

IX.

The end of the early period of cartography.

1520—1550.

Most of the printed maps, during these decennia, were still published as addenda to new editions of Ptolemy's geography. In eleven editions of this work, from 1520 to 1550, which were provided with maps, two hundred and sixty nine of the old maps and two hundred and forty four *tabulæ novæ* were printed, most of them in double folios, while the rest of the map-printing of the same period — if reprints in Münster's cosmography and the small wood-cuts in the works of Bordone and Apianus, are excepted — scarcely amount to one hundred. Thus the cartographical literature of these decennia is still very poor as well as regards its extent as with reference to the composition and execution. Yet the dawn of a new period might even then be discerned, partly from the appearance of new maps, founded on actual topographical investigations, in increasing numbers, partly from the attempts to employ improved methods of projection.

In another respect this period forms an epoch in the development of cartography. A couple of rough wood-cut maps had already been published at Lubeck in 1475, and two highly meritorious editions of Ptolemy, provided with large wood-cut maps, had been published at Ulm in 1482 and 1486, to which may be added some few other more or less important separate maps printed in Germany in the 15th or the first years of the 16th century. With these exceptions, almost all geographical maps had, until 1513, i. e. until the year when the large Strassburg edition of Ptolemy provided with 20 new maps was published, been printed in Italy, although often with the assistance of map-drawers and map-engravers from Gutenberg's fatherland. But from that year the principal seat of the industry of map-printing was transferred to the countries to the north of the Alps, although at first only for a short time. While only a few maps, generally of slight importance, were printed in Italy from 1513 to 1547, by far the greatest part of such works of the next period, 1548—1570, are of Italian origin. From 1570, i. e. from the year when the first edition of *Theatrum Orbis Terrarum* by Ortelius was published, the Netherlands became for a long time the principal seat of map-printing.

The first transfer of this industry to the countries north of the Alps was evidently effected at the expense of the finish of the execution. In this respect the maps of Mercator and Ortelius are the first that can be compared with the old copper-engraved maps from Rome and Venice. Hence the German maps were at first almost exclusively reproduced in a manner little adapted for large cartographical