ELEMENTUM III.

De Quadratis in triangulo non rectangulo, & in parallelogrammo invicem comparatis, & de Quadrilateris circulo inscriptis.

PROPOSITIO I.

546. Theorema. *In omni triangulo obtusangulo* BCD, *si ab angulo acuto* D *perpendicularis* DF *demittatur in latus* BC *productum, & eidem angulo oppositum, erit*

Tab. XII. Fig. 318.

$$\text{I. } \overline{BD}^2 = \overline{CD}^2 + \overline{BC}^2 + 2BC \times CF.$$

$$\text{II. } \overline{BD}^2 = \overline{CD}^2 - \overline{BC}^2 + 2BC \times BF.$$

Euclid. lib. 2, prop. 12.

Demonstratur I. pars. Triangula BFD, CFD sunt rectangula in F.

Ergo $\overline{BD}^2 = \overline{DF}^2 + \overline{BF}^2$

& $\overline{DF}^2 = \overline{CD}^2 - \overline{CF}^2$.

Quia verò BF = BC + CF, erit (n. 508.)

$$\overline{BF}^2 = \overline{BC}^2 + \overline{CF}^2 + 2BC \times CF.$$

Ergo in prima æquatione utrique quadrato $\overline{DF}^2$ & $\overline{BF}^2$ substituendo valorem suum, fiet

$$\overline{BD}^2 = \overline{CD}^2 + \overline{BC}^2 + 2BC \times CF.$$

Quod erat primum.

Demonstratur II. pars. Triangula CF D, BFD rectangula in F, dabunt.

$$\overline{CD}^2 = \overline{DF}^2 + \overline{CF}^2$$

$$\overline{DF}^2 = \overline{BD}^2 - \overline{BF}^2$$

Quia verò CF = BF — EC, erit (n. 509.)

$$\overline{CF}^2 = \overline{BF}^2 + \overline{BC}^2 - 2BC \times BF.$$

Ergo in prima æquatione utrique quadrato $\overline{DF}^2$ & $\overline{CF}^2$ substituendo valorem suum, fiet

$$\overline{CD}^2 = \overline{BD}^2 + \overline{BC}^2 - 2BC \times BF.$$

Adde utrique membro eandem quantitatem $-\overline{BC}^2 + 2BC \times BF$; deletísque terminis se mutuò destruentibus propter signa contraria $\pm$, habebitur

$$\overline{CD}^2 - \overline{BC}^2 + 2BC \times BF = \overline{BD}^2.$$

Quod erat alterum.

PROPOSITIO II.

547. Problema. *In omni triangulo obtusangulo* BCD, *si ab angulo acuto* D *demittatur perpendicularis* DF *in latus eidem oppositum productum, invenire* TAB. XII. Fig. 318.

$$\text{I. } CF = \frac{\overline{BD}^2 - \overline{BC}^2 - \overline{CD}^2}{2BC},$$

II. $BF = \dfrac{\overline{ED}^2 \times \overline{BC}^2 - \overline{CD}^2}{2\,BC}$.

Resolutio & demonstratio. Ex præced. Theor. habes

I. $\overline{BD}^2 = \overline{CD}^2 + \overline{BC}^2 + 2\,BC \times CF$.

II. $\overline{BD}^2 = \overline{CD}^2 - \overline{BC}^2 + 2\,BC \times BF$.

In prima æqualitate adde utrique membro $-\overline{CD}^2 - \overline{BC}^2$, & in secunda adde pariter utrimque $-\overline{CD}^2 + \overline{BC}^2$: erit

I. $\overline{BD}^2 - \overline{BC}^2 - \overline{CD}^2 = 2\,BC \times CF$,

II. $\overline{BD}^2 + \overline{BC}^2 - \overline{CD}^2 = 2\,BC \times BF$.

Harum duarum æqualitatum utrumque dividatur per 2 BC: fiet

I. $\dfrac{\overline{BD}^2 - \overline{BC}^2 - \overline{CD}^2}{2\,BC} = CF$,

II. $\dfrac{\overline{BD}^2 + \overline{BC}^2 - \overline{CD}^2}{2\,BC} = BF$.

Quod erat &c.

Corollarium.

Fig. 318. 548. Hinc ex notis B F, vel C F statim innotescet perpendicularis D F. Nam in triangulo rectangulo B F D, si à quadrato B D subtrahas quadratum B F, vel in triangulo rectangulo C F D, si à quadra-

to

to CD ſubtrahas quadratum CF: in utroque caſu reſiduuum erit quadratum DF, cujus radix quadrata dabit DF perpendicularem quæſitam.

PROPOSITIO III.

549. Theorema. *In omni triangulo ABC, ſi ab angulo A in latus oppoſitum BC demittatur perpendicularis AE, quæ intra triangulum cadat, erit*

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - 2BC \times BE.$$

Euclid. lib. 2. prop. 13.

Demonſtratio. Duo triangula AEC, AEB ſunt rectangula in E. TAB. XII. Fig. 319.

Ergò $\overline{AC}^2 = \overline{AE}^2 + \overline{EC}^2$,

$$\overline{AE}^2 = \overline{AB}^2 - \overline{BE}^2.$$

Et quoniam $EC = BC - BE$, habebitur (n. 509.)

$$\overline{EC}^2 = \overline{BC}^2 + \overline{BE}^2 - 2BC \times BE.$$

His itaque valoribus ſubſtitutis in prima æqualitate, erit

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - 2BC \times BE.$$

Quod erat &c.

Eodem modo demonſtrabitur $\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2 - 2BC \times EC$.

Corollarium. I.

550. In eadem figura ex notis lateribus

bus AC, AB, BC invenietur ſegmentum BE, & conſequenter perpendicularis AE.

Nam (n. 549.) $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - 2BC \times BE$.

Ergò utrique membro æquationis addendo $2BC \times BE$, & utrinque ſubducendo $\overline{AC}^2$, erit

$$2BC \times \overline{BE}^2 = \overline{AB}^2 + \overline{BC}^2 - AC;$$

& utrumque membrum dividendo per 2 BC, fiet

$$BE = \frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2BC}.$$

Invento ſegmento BE, invenies, ut nuper, in triangulo rectangulo ABE perpendicularem AE.

Corollarium II.

551. Hinc habetur dimenſio cujuſcunque trianguli, cujus tria latera ſint nota, licet aream habeat imperviam. Horum quippe Theorematum beneficio innoteſcit perpendicularis, etiamſi eam impedimenta loci non ſinant deſignari. Perpendicularis autem multiplicata per ſemiſſem lateris, producit aream trianguli; ut patet ex dictis.

PROPOSITIO IV.

552. Theorema. *In omni parallelogrammo* ABCD *ſumma duorum quadra-*

dratorum ex diagonalibus AC, BD *æquatur summæ quatuor quadratorum ex lateribus* AB, AD, BC, CD; *hoc est*, TAB. XII. Fig. 320.

$$\overline{AC}^2 + \overline{BD}^2 = \overline{AB}^2 + \overline{AD}^2 + \overline{BC}^2 + \overline{CD}^2.$$

Demonstratio. Ab extremitatibus lateris AD demittantur perpendiculares AE, DF in latus oppositum BC. Constat BE = CF, & consequenter $2BC + BE = 2BC \times CF$.

His positis, triangulum ABC dabit (n. 549.)

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - 2BC \times BE.$$

Et (n. 546.) triangulum BCD dabit

$$\overline{BD}^2 = \overline{BC}^2 + \overline{CD}^2 + 2BC \times CF.$$

Addantur simul hæ duæ æquationes, suppressis terminis æqualibus $-2BC \times BE$, $+2BC \times CF$, qui contrarietate signorum se mutuò destruunt, fiet

$$\overline{AC}^2 + \overline{BD}^2 = \overline{AB}^2 + \overline{BC}^2 + \overline{BC}^2 + \overline{CD}^2;$$

Et uni ex duobus $\overline{BC}^2$ substituitur $\overline{AD}^2$: erit

$$\overline{AC}^2 + \overline{BD}^2 = \overline{AB}^2 + \overline{AD}^2 + \overline{BC}^2 + \overline{CD}^2$$

Quod erat &c.

PROPOSITIO V.

TAB. XII. Fig. 321. 553. Theorema. *Si quadrilaterum* ABCD *circulo sit inscriptum, factum duarum diagonalium* AC, BD *æquatur summæ factorum laterum oppositorum, nimirum,*

$$AC \times BD = AB \times CD + AD \times BC.$$

Demonstratio. Fiat angulus BAP = angulo CAD: erit etiam angulus BAC = angulo DAP. His positis.

I. Triangula BAP, CAD erunt æquiangula, & similia. Nam angulus BAP = angulo CAD per Constr., & angulus ABP = angulo ACD. Quare AB : AC :: BP : CD; & consequenter (n. 379.)

$$AC \times BP = AB \times CD.$$

II. Duo triangula CAB, DAP sunt pariter similia. Nam præter angulum BAC = angulo DAP, erit etiam angulus ACB = angulo ADP.

Ergo $AC : AD :: BC : PD$;

atque adeo $AC \times PD = AD \times BC$.

Utriusque æquationis membra invicem addantur: prodibit $AC \times BP + AC \times PD = AB \times CD + AD \times BC$, hoc est, quia $BP + PD = BD$,

$$AC \times BD = AB \times CD.$$

Quod erat &c.

PRO-

PROPOSITIO VI.

TAB. XII. Fig. 321.

554. Theorema. *In quadrilatero* AB CD, *quod circulo ſit inſcriptum, ſi ducantur diagonales* AC, BD, *diagonalis* AC *aliam* BD *ſecabit in partes* BE, DE *proportionales factis* AB × BC, AD × DC *laterum huic diagonali adjacentium, nimirum,*

BE : DE : : AB × BC : AD × DC.

Demonſtratio. I. Triangula AEB, DEC ſunt ſimilia; nam angulus BAE = angulo CDE. Itaque

BE : CE : : AB : CD.

II. Triangula BEC, AED ſunt pariter ſimilia.

Ergò CE : DE : : BC : AD.

Harum itaque duarum proportionum terminis reſpectivè multiplicatis, ſuppreſſóque termino CE, qui invenitur in primo antecedente, & primo conſequente, prodibit BE : DE : : AB × BC : AD × CD. Quod erat &c.

GEO-